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Wave component solutions of free vibration and mode damping loss factor of finite length periodic beam structure with damping material

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ABSTRACT

Free vibration and mode damping loss factor of the finite periodic beam structure with viscoelastic damping material are investigated in this paper based on wave component method. A simple iterative operation for the natural vibration modes of the beam structure is firstly proposed by considering that the mode shape can be formed through superposition of the wave shapes of the wave components of the beam structure at the corresponding natural frequency. And then, wave component solution of the mode damping loss factor of the beam structure is obtained and analyzed based on the modal stain energy method. It is found that the kinetic energy of the unit cell of propagative wave component is always the same with the strain energy, and for non-propagative wave component these two quantities will never be equal. In numerical examples, the proposed iterative operation is validated.

1. Introduction

Cellular lattice core sandwich structure has always been the hot spot of the engineering application and scientific research over the years because of its strong potential in multifunctional design aspects, such as lightweight, heat insulation, shock and vibration reduction [1]. The vibration reduction design of periodic structure can be carried out based on two sets of data: band gap characteristic, accurate vibration response, respectively. For infinite periodic structure, band gap analysis can be carried out to obtain the vibration suppression characteristic of the structure. For finite periodic structure, since the influence of boundary condition is strong and hence the vibration reduction design should be carried out based on the accurate response.

For simple periodic beam and plate structure with regular geometry, since the dynamic equilibrium equation of the unit cell and even the whole structure can be derived analytically [2–4], the vibration response can be obtained with high efficiency and high accuracy. However, it is difficult to establish precise analytical dynamic governing equations for periodic structures with advanced lightweight materials, such as cellular materials, which generally have complex geometry configurations. In the vibration analysis of periodic structures with cellular materials, a frequently adopted procedure is to obtain homogenization material properties on the unit cell scale firstly [5–7] and then to calculate the vibration of the structure based on the homogenization material properties by using analytical or discretized method such as FEM [8]. Although this procedure is efficient for structures with complex material geometry configuration, it can only be used to analyze the vibration of structures when structural wave length is larger than the size of the unit cell [9,10]. Another solution method for the vibration analysis of structure is based on the full-size finite element model, and for undamped structure the vibration response can be calculated efficiently by using the mode superposition method [8]. For structures with damping materials, direct solution based on the physical matrices, i.e. the stiffness matrix, the mass matrix and the damping matrix of the structure, would generate huge computations; solution based on the mode superposition method would require the natural frequency, mode shape and mode damping loss factor of the structure. At present, the natural frequency and the mode shape can be generally accurately obtained from commercial software. However, it is difficult for commercial software to give directly the mode damping loss factor. Therefore, it is of significance to develop efficient and accurate prediction method for mode damping loss factor of composite structures. Modal strain energy method is often used to obtain the mode damping loss factor of composite structures [11]. Maheri and Adams [12] calculate the mode damping loss factor of free composite laminated plate based on the finite element method and modal strain energy theory, and the theoretical results agree well with the experimental results. The mode damping loss factor and forced vibration response of a variety of composite structures are calculated by Berthelot et al. [13] utilizing the finite element method and the modal strain energy method, and the accuracy of the result is validated experimentally. Yang et al. [14] prepare pyramid core lattice composite sandwich plate and discuss the

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influence of damping materials on the mode damping loss factor based on the finite element method and the modal strain energy method. These works need to solve the eigenvalue problem of free vibration, dissipation energy and strain energy of the whole structure. However, with the increasing of the model size or the parameter variability of the structure, huge amount of calculation would be encountered. Modal testing is another way to obtain the structural mode damping [15]. However, extensive preparation and experimentation at the design stage would result in significant costs of time and manpower. The vibration response of periodic cellular material structure can also be calculated by wave propagation method [16–22]. At this point, the contributions of the high order near-field wave components need to be fully considered. However, for complex geometry configurations, the near-field wave components of high order are difficult to be obtained with enough accuracy.

By considering that the beam structure investigated in this paper is periodic in material and geometry, the wave component method is used here to evaluate the free vibration and the mode damping loss factor of the finite length periodic beam structure. The wave component method is based on analysis of just a unit cell of the beam structure, and hence would be more efficient than the full size finite element analysis especially when the beam structure is consist of lots of unit cells. The wave components of the beam structure are obtained through dispersion analysis based on the dynamic modeling of the unit cell of the structure. A simple iterative solution for the natural frequency and mode shape of the beam structure is established based on the physical relation between the mode shape and the wave components at the corresponding natural frequency [23,24]. And then the mode damping loss factor of the beam structure is calculated in terms of superposition of the wave components. Since the mode damping loss factor of the whole beam structure is obtained based on the dynamic modeling of the unit cell, the computation cost is not influenced by the size and the boundary condition of the whole structure. At last, an anti-tetrachiral core sandwich beam with damping material is used as the example structure to illustrate the present iterative operation and the wave component method.

2. Wave propagation description of the vibration of periodic beam structure

As shown in Fig. 1, the periodic beam structure is constructed by a series of identical unit cell along the wave propagation direction. The length of each unit cell is Δ. The conditions of displacement compatibility and force balance are satisfied at the coupling edges of each two adjacent unit cells (assuming that the left and the right coupling edges of each unit cell have the same node distribution, i.e. have the same degrees of freedom). The unit cell of the beam structure can be of arbitrary complex configuration. The steady state vibration of the beam structure can be seen as superposition of pairs of positively travelling wave components and negatively travelling wave components. The vibration characteristics of finite length beam structure with complex material configuration are significantly affected by the boundary condition. At this point, the mode shape of the beam structure cannot be formed by a single waveform. Multiple wave components are needed [24]. The ith mode shape of the beam structure $\mathbf{A}_i$ can be expressed as $\mathbf{A}_i = \mathrm{Re}\{\boldsymbol{\eta}\mathbf{a}\}$, where $\boldsymbol{\eta} = [\boldsymbol{\eta}^+ \ \ \boldsymbol{\eta}^-]$ is the displacement part of the wave shape matrix and $\mathbf{a} = \begin{Bmatrix}\mathbf{a}^+\\ \mathbf{a}^-\end{Bmatrix}$ is the amplitude vector of the wave components. To avoid numerical ill-conditioning problem due to the simultaneous maximum and minimum values of the wave amplitudes, the amplitudes of the wave components at the left edge of the nth unit cell are expressed by the amplitudes of the positively travelling wave components at the left edge of the beam structure $\mathbf{a}_L^+$ and the amplitudes of the negatively travelling wave components at the right edge of the beam structure $\mathbf{a}_R^-$, i.e. $\begin{Bmatrix}\mathbf{a}^+\\ \mathbf{a}^-\end{Bmatrix} = \boldsymbol{\Lambda}\begin{Bmatrix}\mathbf{a}_L^+\\ \mathbf{a}_R^-\end{Bmatrix}$. Here $\boldsymbol{\Lambda} = \mathrm{diag}\{\mathrm{diag}\{\mathrm{e}^{-\mathrm{i}k_i(n-1)\Delta}\}, \mathrm{diag}\{\mathrm{e}^{-\mathrm{i}k_i(N-n+1)\Delta}\}\}$, k_i is the wave number of the ith positively travelling wave component and N is the total number of the unit cell.

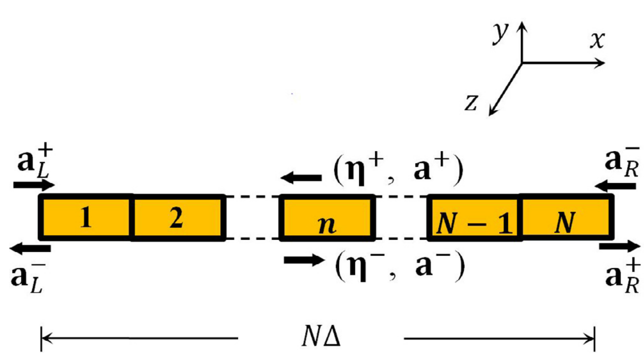

Fig. 1. Schematic diagram of periodic beam structure and wave propagation.

3. Free vibration: natural frequency and mode shape

From $\mathbf{A}_i = \mathrm{Re}\{\boldsymbol{\eta}\mathbf{a}\}$, the relative amplitudes of the wave components are required to determine the mode shape of the beam structure. The wave components must meet the boundary conditions of the beam structure. As shown in Fig. 2, there have $\mathbf{E}_L\begin{Bmatrix}\mathbf{q}_L\\ \mathbf{f}_L\end{Bmatrix} = 0$ and $\mathbf{E}_R\begin{Bmatrix}\mathbf{q}_R\\ \mathbf{f}_R\end{Bmatrix} = 0$ at the left and the right edges of the unit cell, respectively. Here $\mathbf{E}_L$ and $\mathbf{E}_R$ are indicating matrices of boundary condition, for example, $\mathbf{E}_L = \mathrm{diag}\{0, 0, ..., 0, 1, 1, ..., 1\}$ for free boundary condition. $\mathbf{q}_L$, $\mathbf{q}_R$, $\mathbf{f}_L$ and $\mathbf{f}_R$ are displacement vectors and internal force vectors at the left and the right edges of the unit cell. Since the displacement and the internal force at the left and the right edges of the unit cell can be expressed in terms of the superposition of the wave components, then there have $\mathbf{E}_L\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_L^+\\ \mathbf{a}_L^-\end{Bmatrix} = 0$ and $\mathbf{E}_R\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_R^+\\ \mathbf{a}_R^-\end{Bmatrix} = 0$, where $\boldsymbol{\Phi}$ is the wave shape matrix of the unit cell. The weighted residual operation is further adopted. Pre-multiplying both sides of $\mathbf{E}_L\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_L^+\\ \mathbf{a}_L^-\end{Bmatrix} = 0$ and $\mathbf{E}_R\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_R^+\\ \mathbf{a}_R^-\end{Bmatrix} = 0$ by $\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{J}$ gives

$$\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{J}\mathbf{E}_L\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_L^+\\ \mathbf{a}_L^-\end{Bmatrix} = 0, \quad \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{J}\mathbf{E}_R\boldsymbol{\Phi}\begin{Bmatrix}\mathbf{a}_R^+\\ \mathbf{a}_R^-\end{Bmatrix} = 0 \tag{1}$$

The detail illustration of $\boldsymbol{\Phi}$ and $\mathbf{J}$ can be found in the following Section 5.

The reflection relations between the wave components at the two ends of the beam structure can be further obtained from Eq. (1)

$$\mathbf{a}_L^+ = \mathbf{R}_L\mathbf{a}_L^- = \mathbf{R}_L\mathbf{T}(b_0)\mathbf{a}_R^-, \quad \mathbf{a}_R^- = \mathbf{R}_R\mathbf{a}_R^+ = \mathbf{R}_R\mathbf{T}(b_0)\mathbf{a}_L^+ \tag{2}$$

Eq. (2) further gives

$$(\mathbf{I}-\mathbf{R}_L\mathbf{T}(b_0)\mathbf{R}_R\mathbf{T}(b_0))\mathbf{a}_L^+ = 0 \tag{3}$$

where $\mathbf{R}_L$ and $\mathbf{R}_R$ which can be obtained from Eq. (1) are wave reflection coefficient matrices at the left and the right boundary edge of the beam structure, respectively, $\mathbf{T}(x) = \mathrm{diag}\{\mathrm{e}^{-\mathrm{i}k_ix}\}$ is the wave propagation matrix and $\mathbf{I}$ is unit matrix. The strategy used to derive Eqs. (1)–(3) can also be found in Ref. [18].

The relative amplitude of the wave components can be obtained

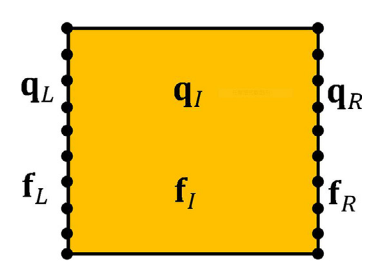

Fig. 2. Schematic diagram of the unit cell of the periodic structure.