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An extended Ritz formulation for buckling and post-buckling analysis of cracked multilayered plates

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ABSTRACT

An extended Ritz formulation for the analysis of buckling and post-buckling behaviour of cracked composite multilayered plates is presented. The formulation is based on: (i) the First-order Shear Deformation Theory to model the mechanics of the multilayered plate; (ii) the von Kármán's theory to account for geometric nonlinearities; (iii) the use of an extended set of approximating functions able to model the presence of an embedded or edge crack and to capture the crack opening fields as well as the global behaviour within a single cracked domain. The numerical results of the buckling analyses and the equilibrium paths in the post-buckling regime are compared with the results from finite elements simulations, confirming the accuracy and potential of the formulation.

1. Introduction

Multilayered composite plates are widely and effectively employed in a broad range of structural engineering applications. With respect to metallic structures, they provide the engineer with a wider set of design parameters [1] such as the stacking sequence, the layers' thickness and, more recently, the possibility of spatially varying the fibers orientation within a single layer using variable angle tow technologies [2,3]. Thanks to their high stiffness-to-weight and strength-to-weight ratios, composite plates are typically used as structural components in thin-walled structures that have to comply with lightweight requirements, e.g. in the aerospace sector; for such a reason, they can undergo buckling and post-buckling deformations [4], which must be reliably predicted during the design process. This is particularly relevant when the design of thin-walled structures must also comply with damage tolerance provisions and therefore the presence of damage must be considered. In fact, the presence of cracks in thin plates can induce non-trivial mechanical behaviours: as an example, a crack in a thin plate subjected to tensile stress can induce local buckling around the crack tips and significant amplification of the stress intensity factors [5] as well as different non-trivial fracture opening modes.

The analysis of thin-walled structures with multilayered composite plates generally requires the use of numerical models, due to the complex interactions between various design parameters and the difficulty in obtaining closed form solutions for general boundary conditions. The Finite Element Method (FEM) represents the standard technique employed to model buckling and post-buckling behaviour of undamaged and damaged composite plates. However, finite elements solutions heavily rely on the quality on the employed mesh, which must be carefully generated in order to conform to the geometry and capture the gradients of the unknown fields. These aspects are even more critical in fracture mechanics problems.

In the literature, a technique developed to overcome some drawbacks of standard FEMs is the Extended Finite Element Method (XFEM), which was originally proposed by Belytschko and coworkers [6,7] and then successfully employed by many researchers to address, besides crack problems in two- and three-dimensional bulk domains [8–10], fracture mechanics problems in thin-walled structures. Examples are: fracture mechanics of isotropic Mindlin plates [11]; crack propagation coupled to cohesive zone modelling in non-linear thin shells [12]; fracture in thin shells using an extended isogeometric approach [13]; buckling analyses of cracked isotropic [14] and composite [15] plates and frequency analyses of cracked functionally graded plates [16]. XFEM is formulated within a FEM framework and assumes the unknown fields to be written as a sum of standard contributions, which correspond to the classic finite elements approximation based on elemental shape functions and unknown nodal values, and enrichment contributions, which account for the presence of the (crack) discontinuity. By suitably choosing the enrichment functions, the unknown fields affected by the presence of a crack or discontinuity can be accurately modelled. A typical example is provided by a cracked solid modelled within the linear fracture mechanics hypotheses; in this case,

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the set of enrichment functions are based on the theoretical asymptotic solution, i.e. on the square root behaviour of the displacement field with respect to the distance from the crack tip (see e.g. [17]). Although the mesh need not conform to the crack morphology, XFEM studies usually employ smaller mesh elements in the vicinity of the crack tip in order to ensure an accurate solution and avoid an excessive number of degrees of freedom.

As an alternative to finite elements approaches and with the aim of simplifying data preparation and meshing efforts, several researchers have proposed numerical models based on the Ritz method. In Ritz approaches, the sought unknown fields are expanded in series using a set of suitably defined functions, whose coefficients represent the unknowns of the discrete problem. The accuracy and versatility of Ritz-based numerical models has been demonstrated in studies dealing with the frequency and buckling response of both thin and thick plates [18–21] and with the post-buckling behaviour of thick plates [22]. Ritz-based models have also been used for studying the buckling and post-buckling behaviour of thin plate assemblies [23–25] and the free vibrations and buckling modes of stiffened shells [26,27].

To include the presence of a crack, the Ritz method has been employed in combination with the multi-region approach. In the literature, examples are available for free vibrations [28–32], buckling analyses [33,34] and post-buckling response [35–38] of plates and stiffened plates. Single-domain Ritz-based formulations are very rare in the literature and are limited to linear analyses of free vibrations and buckling modes of cracked plates [39–42]. To the best of the Authors' knowledge, single-domain formulations for non-linear analysis of cracked multilayered composite plates are not available in the literature.

This works presents a novel single-domain formulation based on the First-order Shear Deformation Theory (FSDT) and the von Kármán's theory hypotheses [43] to model the buckling and post-buckling behaviour of cracked multilayered composite plates. The formulation, termed X-Ritz, combines the advantages of both the Ritz method and the XFEM strategy, as it is obtained by enriching the Ritz series expansion with suitably defined crack functions that allow to resolve the presence of either an embedded crack or an edge crack in multilayered quadrangular plates. As a consequence, it has the advantage of a very simple preprocessing stage, as it only requires the geometrical information on the plate and the crack. A broad campaign of numerical tests verifies its accuracy and flexibility.

The paper is organised as follows: Section 2 introduces the geometrical features of cracked multilayered composite plates as well as the notation to model their non-linear behaviour under the hypotheses of the FSDT and von Kármán's theory; Section 3 describes the proposed single-domain extended Rayleigh-Ritz formulation; Section 4 collects and discusses the performed numerical tests aimed at verifying the accuracy of the proposed formulation for both buckling and post-buckling analysis of cracked plates; eventually, Section 5 draws some conclusions.

2. Notation and basic equations

Let us consider a cracked planar quadrilateral composite plate referred to a Cartesian coordinate system O_{xyz}, with the x- and y-axes parallel to the plate's plane and the z-axis along the plate's thickness. In this reference system, the plate occupies the volume $\Omega \times [h_b, h_t]$ where $z = h_b$ and $z = h_t$ denote the heights of the bottom and top surfaces of the plate, respectively, and Ω is the projection of the plate onto the plane $z = 0$, which is usually referred to as the modelling plane. The domain Ω is defined by the position of its four vertices $\boldsymbol{V}^{(\alpha)} = \{V_x, V_y\}^{(\alpha)}$, $\alpha = 1, \ldots, 4$ and by the two crack tip points $\boldsymbol{T}^{(1)}$ and $\boldsymbol{T}^{(2)}$ as shown in Fig. 1.

An *embedded* crack is identified by the points $\boldsymbol{T}^{(1)}$ and $\boldsymbol{T}^{(2)}$ located within the quadrilateral plate domain, whereas an *edge* crack corresponds to one point located within the domain and the other one lying on one of the plate edges, as shown in Figs. 1b and c respectively. To simplify the expression of the enrichment crack functions, it is convenient to define a polar coordinates system $\{r_1, \theta_1\}$ centered at the tip of the edge crack, see Fig. 1c, or two polar coordinates systems $\{r_1, \theta_1\}$ and $\{r_2, \theta_2\}$ centered at the tips of the embedded crack, see Fig. 1b.

On the other hand, as shown in Fig. 1a, the multilayered plate section consists of L laminae stacked together in such a way that the bottom surface of the k-lamina lying at $z = h_b^{\langle k \rangle}$ coincides with the top surface of the $(k-1)$-lamina lying at $z = h_t^{\langle k-1 \rangle}$. The generic k-th lamina is assumed orthotropic and having one material axis aligned with the plate's z-axis and the other two axes generally oriented with respect to the x- and y-axis by means of the orientation angle $\theta^{\langle k \rangle}$, which coincides with the fibers' direction.

According to the First-order Shear Deformation Theory, the displacement field $\boldsymbol{d} = \boldsymbol{d}(x, y, z)$ in the reference system O_{xyz} is expressed in terms of the generalized displacements u, v, w, ϑ_x and ϑ_y by [4]

$$d_x(x, y, z) = u(x, y) + z\vartheta_x(x, y) \tag{1a}$$

$$d_y(x, y, z) = v(x, y) + z\vartheta_y(x, y) \tag{1b}$$

$$d_z(x, y, z) = w(x, y) + \overline{w}(x, y) \tag{1c}$$

where u, v, w are the components of the displacement field of a point on the modeling plane; ϑ_x, ϑ_y are the rotations of a segment normal to the modeling plane about the y- and x-axis, respectively; and $\overline{w}$ accounts for an initial imperfection of the plate. Eqs. (1) can be written in compact form, see e.g. [35], as follows

$$\boldsymbol{d}(x, y, z) = \boldsymbol{u}(x, y) + z\boldsymbol{\vartheta}(x, y) + \overline{\boldsymbol{u}}(x, y) \tag{2}$$

where $\boldsymbol{u} \equiv \{u, v, w\}^T$, $\boldsymbol{\vartheta} \equiv \{\vartheta_x, \vartheta_y, 0\}^T$ and $\overline{\boldsymbol{u}} = \{0, 0, \overline{w}\}^T$.

The plate deformation is expressed by means of the Green-Lagrange strain tensor, whose components are suitably partitioned into *in-plane* components $\boldsymbol{e}_p \equiv \{e_{xx}, e_{yy}, e_{xy}\}^T$ and *out-of-plane* components $\boldsymbol{e}_n \equiv \{e_{xz}, e_{yz}, e_{zz}\}^T$. According to the von Kármán's assumptions, the retained non-linear terms of strains are those involving the derivatives of the displacement component d_z with respect to the in-plane variables x and y [4]. Following Milazzo and Oliveri [35] and remembering that the initial imperfection is assumed small, $\boldsymbol{e}_p$ and $\boldsymbol{e}_n$ can then be written in terms of the generalized displacements introduced in Eq. (1) using the following compact notation

$$\boldsymbol{e}_p = \mathscr{D}_p\boldsymbol{u} + \frac{1}{2}(\mathscr{D}_p \otimes \ w)\mathscr{D}_n\boldsymbol{u} + (\mathscr{D}_p \otimes \ \overline{w})\mathscr{D}_n\boldsymbol{u} + z\mathscr{D}_p\boldsymbol{\vartheta} = \boldsymbol{\varepsilon} + \widetilde{\boldsymbol{\varepsilon}} + z\boldsymbol{\kappa} \tag{3a}$$

and

$$\boldsymbol{e}_n = \mathscr{D}_n\boldsymbol{u} + \boldsymbol{\vartheta} + \mathscr{D}_n\overline{\boldsymbol{u}} = \boldsymbol{\gamma} + \overline{\boldsymbol{\gamma}}, \tag{3b}$$

respectively, where the symbol $\otimes$ denotes the Kronecker product and the following generalized strain fields have been defined:

$$\boldsymbol{\varepsilon} = \mathscr{D}_p\boldsymbol{u} + (\mathscr{D}_p \otimes \ \overline{w})(\mathscr{D}_n\boldsymbol{u}), \tag{4a}$$

$$\boldsymbol{\kappa} = \mathscr{D}_p\boldsymbol{\vartheta}, \tag{4b}$$

$$\widetilde{\boldsymbol{\varepsilon}} = (\mathscr{D}_p \otimes \ w)(\mathscr{D}_n\boldsymbol{u})/2, \tag{4c}$$

$$\boldsymbol{\gamma} = \mathscr{D}_n\boldsymbol{u} + \boldsymbol{\vartheta} \tag{4d}$$

$$\overline{\boldsymbol{\gamma}} = \mathscr{D}_n\overline{\boldsymbol{u}}; \tag{4e}$$

with

$$\mathscr{D}_p = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ \partial/\partial y & \partial/\partial x & 0 \end{bmatrix} \quad \text{and} \quad \mathscr{D}_n = \begin{bmatrix} 0 & 0 & \partial/\partial x \\ 0 & 0 & \partial/\partial y \\ 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

It is worth noting that, according to Eqs. (3a) and (3b), the shear components of $\boldsymbol{e}$, i.e. e_{xy}, e_{xz} and e_{yz}, are twice the components of the Green-Lagrange strain tensor as defined classically. The mechanical variable associated to the Green-Lagrange strain tensor is the second