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Theoretical solutions of 2–2 multi-layered cement-based piezoelectric composite under impact load

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ABSTRACT

An analysis model of the dynamic properties of 2–2 multi-layered cement-based piezoelectric composite under impact load and its definite problems are investigated based on the theory of piezo-elasticity in this paper. The theoretical solutions are obtained according to the Duhamel integral and variable separation method. After analyzing the physical meaning and the convergence of the theoretical solutions, the numerical simulations of the composite are presented by utilizing the various loads including the transient step, the transient haversine wave and the transient isosceles triangle loads. The theoretical solutions are compared with the numerical simulations, as well as reported results. Excellent agreements among these results are found. In addition, the influences of the number of layers of the composite and the piezoelectric parameters on the response of dynamic properties are discussed. The work could benefit the design and preparation of the 2–2 multi-layered cement-based piezoelectric composite in engineering practice.

1. Introduction

Piezoelectric composites, especially cement-based piezoelectric composites, have extraordinary sensitive transduction and compatibility characteristics with most traditional materials of construction such as concrete and cement in civil engineering [1]. Cement-based piezoelectric composites consist of a piezoelectric phase and a cement matrix based on the different volume fraction and the composite rule. It is essential to study the overall performance of the cement-based piezoelectric composites for the practical engineering optimization and application, such as the transducers design [2,3].

Most of the reported researches have concentrated on fabricating the cement-based piezoelectric composites and determining the material parameters and properties of the composites experimentally. Li et al. [4] and Zhang et al. [5] have fabricated 2–2 cement-based piezoelectric composites, and presented the piezoelectric and converse piezoelectric effects of the composites regarding to the sensor and actuator attributes. Huang et al. [6,7] have studied the influence of volume fraction of the piezoelectric phase and the water-cement ratio on the properties of the 2–2 cement based piezoelectric composites. Cheng et al. [8] have fabricated the 2–2 cement-based piezoelectric composites by dice-and-fill technique and studied the influences of the composite thickness on the piezoelectric, dielectric and electromechanical properties of the composite. Chaipanich and Jaitanong [9] have fabricated the lead zirconium titanate-cement-encapsulated carbon composites based on the pressed-cured method and investigated their dielectric properties. By using a dice and fill process, Chaipanich et al. [10] have also fabricated 2–2 connectivity lead magnesium niobate-lead titanate/cement composites and investigated their dielectric properties. The results of these researches have a guiding significance for the study of cement-based piezoelectric composites. Lu et al. [11] have firstly combined the cement-based piezoelectric composite with the home-programmed DEcLIN monitoring system to examine the concrete fracture process in mortars. By using the mixing and compressing process, Hunpratub et al. [12] have prepared the 0–3 cement-based piezoelectric composites with different material volume fractions. In their paper, the influences of the piezoelectric particle size on the piezoelectric and dielectric characteristics of composites were investigated. The results show that the piezoelectric and dielectric properties of the composites are different when the size and volume fraction of the piezoelectric particle differ. Jaitanong et al. [13] have produced the 0–3 lead zirconium titanate ceramic-cement composites with specific volume fraction of the piezoelectric phase and studied the hysteresis properties of the composites under the compressive stress. Xu et al. [14] have used the lead zirconium titanate, cement powder and epoxy resin as the raw materials to design and fabricate the 2–2 cement/polymer based piezoelectric composites, and studied the effects of the composite thickness and filler content on the composite

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characteristics. They also studied the piezoelectric, dielectric, electro-mechanical coupling characteristics and coupling effects of the composite [15].

Meanwhile, the theoretical studies on the characteristics of cement-based piezoelectric composites, especially on the dynamic characteristics, attracted more attentions in recent years but are still limited. By utilizing the precise integration algorithm and the dual vector form method, Liu et al. [16] have studied the elastodynamic axisymmetric behaviors and characteristics of the transversely isotropic piezoelectric medium with multiple layers. Zhang and Shi [17] have obtained the analytical solutions of the expansion and contraction types of actuators in case of the external electric potential and shearing load based on the airy stress function method and the theory of piezo-elasticity. Yao and Shi [18] have studied the vibration of a piezoelectric beam under different loads by using the differential quadrature method. Shi and Wang [19] have obtained the analytical solutions of four kinds of 2–2 cement-based piezoelectric composites under the external harmonic load based on the theory of piezo-elasticity and the displacement method. By utilizing the theory of elasticity and the Airy stress function method, Zhang [20] has obtained the theoretical solutions of the electrical field and mechanical field of the 2–2 cement-based piezoelectric curved composites and discussed the effects of several parameters on the properties of the composites. Bodaghi and Shakeri [21] have investigated the dynamic characteristics of the functionally graded piezoelectric cylindrical panels under the impulsive loads by using the Hamilton's principle, the Fourier series expansions and several other mathematical methods. Based upon the theory of impact mechanics and piezo-elasticity and by using the traveling and standing wave methods, Zhang and Ma [22] have obtained the exact solutions of the electrical and mechanical fields of the piezoelectric structure under the impact load without considering the perturbation and noise. The dynamic load could cause serious damage to the engineering structures, particularly the composite structures. It is thus of practical significance to study the dynamic characteristics of the composite structures under the impact load.

To illustrate the effect of impact load and specific material parameters on the dynamic properties of the 2–2 multi-layered cement-based piezoelectric composite, an exact analysis model and its definite problems are presented in Section 2 based on the theory of piezo-elasticity. In Section 3, the theoretical solutions of the 2–2 multi-layered cement-based piezoelectric composite under the impact load are obtained according to the Duhamel integral and variable separation method. Meanwhile, the applied impact load and the physical meaning of the solution are further discussed. In Section 4, the convergence properties of the theoretical solution are analyzed. Moreover, the numerical simulations of the composite are presented by utilizing the transient step load, the transient haversine wave load and transient isosceles triangle load, respectively. The theoretical solutions are compared with the numerical simulations, as well as Zhang's results [22] and Li's results [23]. Excellent agreements are found. In addition, the displacement, stress and electric potential distribution of the composites with different layers are studied, and the influences of the number of layers of the composite and the piezoelectric parameters on the responses of structural displacement, electric potential and fixed end stress amplitudes are discussed.

2. Basic equations

The schematic of the 2–2 multi-layered cement-based piezoelectric composite under study is shown in Fig. 1(a), with one end free and other fixed. The free end of the composite is subjected to an impact load $\delta(t)$. We use P and C to denote the piezoelectric layer and cement layer, individually. The number of layer of the piezoelectric and cement layer, individually, are N and $N+1$. The symbols of the material parameters of the $P\#i$ ($i = 1,2,\cdots,N$) and $C\#i$ ($i = 1,2,\cdots,N+1$) are listed in Table 1. Unless otherwise specified, the provision for the index of i applies throughout the paper and thus omitted. The thicknesses of the i-th cement layer and i-th piezoelectric layer are deduced by $h_{2i-1} = l_{2i-1}-l_{2i-2}$ and $h_{2i} = l_{2i}-l_{2i-1}$, respectively. For different cement layers and piezoelectric layers, the material parameters in Table 1 are not necessarily the same. Referring to the rectangular Cartesian coordinate system, the symbols of the mechanical quantities of the $P\#i$ and $C\#i$ are shown in Table 2. The elemental force analysis of the longitudinal vibration of the composite is shown in Fig. 1(b).

The equilibrium equation of the composite in Fig. 1(a) can be obtained as follows based on the Newton's second law and Fig. 1(b):

$$\rho A dz\frac{\partial^2 w}{\partial t^2} = \frac{\partial F_N}{\partial z}dz + \delta(t)\delta(z-l_{2N+1})Adz \tag{1}$$

here w is the displacement and ρ, A are the density and area of the cross section of the specific piezoelectric and cement layer (the index is omitted here for conciseness). $\delta(t)\delta(z-l_{2N+1})$indicates that the free end of the composite is subjected to the impact load $\delta(t)$. $\delta(t)$ and $\delta(z-l_{2N+1})$ can be expressed as:

$$\delta(t) = \begin{cases}\infty, & t = 0\\ 0, & t \neq 0\end{cases} \tag{2a}$$

$$\delta(z-l_{2N+1}) = \begin{cases}\infty, & z = l_{2N+1}\\ 0, & z \neq l_{2N+1}\end{cases} \tag{2b}$$

From Eq. (1), we have:

$$\frac{\partial^2 w}{\partial t^2} = \frac{1}{\rho}\frac{\partial \sigma_N}{\partial z} + \frac{\delta(t)\delta(z-l_{2N+1})}{\rho} \tag{3}$$

The first term on the right side of Eq. (3) indicates the acceleration of the composite generated by the internal force of the piezoelectric layers or cement layers, and the second term indicates the acceleration produced by the external impact load upon the whole composite.

Based on the principle of material composite [24], the theoretical density ρ_T of the 2–2 multi-layered cement-based piezoelectric composite could be denoted as:

$$\rho_T = \sum_{i=1}^{N}\rho_{pi}f_{pi} + \sum_{i=1}^{N+1}\rho_{ci}f_{ci} \tag{4}$$

The combination of Eqs. (3) and (4) leads to:

$$\frac{\partial^2 w}{\partial t^2} = \frac{1}{\rho}\frac{\partial \sigma_N}{\partial z} + \frac{\delta(t)\delta(z-l_{2N+1})}{\rho_T} \tag{5}$$

For $C\#i$, by using Eq. (5), the basic equations can be written as follows without considering the body charge and body force:

$$\begin{aligned}&\frac{\partial^2 w_{ci}}{\partial t^2} = \frac{1}{\rho_{ci}}\frac{\partial \sigma_{ci}}{\partial z} + \frac{\delta(t)\delta(z-l_{2N+1})}{\rho_T}\\ &\sigma_{ci} = C_{33ci}\varepsilon_{ci}\\ &\varepsilon_{ci} = \frac{\partial w_{ci}}{\partial z}\end{aligned} \tag{6}$$

Eq. (6) can also be expressed as:

$$\frac{\partial^2 w_{ci}}{\partial t^2} - C_{ai}^2\frac{\partial^2 w_{ci}}{\partial z^2} = \frac{\delta(t)\delta(z-l_{2N+1})}{\rho_T} \tag{7}$$

where $C_{ai} = \sqrt{C_{33ci}/\rho_{ci}}$ denotes the propagation speed of elastic wave in the $C\#i$. Eq. (7) is the control equation of the $C\#i$.

For $P\#i$, by using Eq. (5), the basic equations can be written as follows without considering the body charge and body force:

$$\begin{aligned}&\frac{\partial^2 w_{pi}}{\partial t^2} = \frac{1}{\rho_{pi}}\frac{\partial \sigma_{pi}}{\partial z} + \frac{\delta(t)\delta(z-l_{2N+1})}{\rho_T}\\ &\sigma_{pi} = C_{33pi}\varepsilon_{pi} - e_{33i}E_i\\ &\varepsilon_{pi} = \frac{\partial w_{pi}}{\partial z}\end{aligned} \tag{8}$$