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Dynamic stability of sandwich functionally graded micro-beam based on the nonlocal strain gradient theory with thermal effect

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ABSTRACT

In this study, the dynamic stability of functionally graded (FG) size dependent sandwich micro-beam subjected to parametric axial excitation with different boundary conditions including thermal effects is investigated. Based on the nonlocal strain gradient theory (NLSGT) in conjunction with the first order shear deformation beam theory (FSDBT) and Hamilton's principle, governing equations of motion and corresponding boundary conditions were obtained. Differential quadrature (DQ) method is utilized to solve the derived differential equations. Material properties of the FG part of sandwich micro-beam are varied through the thickness of the beam by assuming the classical rule of mixture. Effects of the slenderness ratio (L/h), nonlocal parameter (ea), FG mixture index (k), length scale parameter (l_m), static load factor (η_s), temperature change, various boundary (C-C), (S-S), (C-S) conditions and different cross-section shapes on the dynamic stability behavior of the sandwich micro-beam are investigated. Numerical solution for determining the parametric instability regions of a FG sandwich micro-beam under different boundary conditions and various effects are the original contributions of this study.

1. Introduction

In recent years number of studies intensified on the so called functionally graded materials (FGMs), a novel type of composites. These inhomogeneous materials generally consist of two different type of materials and formed in a way that properties varies continuously through spatial directions. Both mechanical and physical properties of these continuous composite structures are enhanced as they inherit the desirable properties of the main materials and mitigate the undesirable properties. One common application is using metal and ceramic together in this fashion and the resulting FG composite product has both enhanced strength and thermal resistance together when compared to metal (high strength-low thermal resistance) and ceramic (high thermal resistance-low strength) only configurations. Varying the properties continuously also helps for a better structural integrity of the FG composite as problem of mismatch of material properties due to abrupt changes is mitigated as well. These superior qualities make FGMs preferable for many high-tech industries such as aerospace, automotive, nuclear, and defense. Thus understanding mechanical characteristics of FGMs in static and dynamic conditions has been focus of many research projects. Efforts for understanding buckling and vibration of these FG structures resulted in some theoretical models. Also there have been efforts for addressing the mechanical performance of FG structures. [1–19]

Another type of structures which is commonly used in many engineering applications are sandwich structures due to their high strength to weight ratio. Common sandwich structures generally consist of a soft core which is bonded to two skins which are usually made from tougher materials. The abrupt change in material properties of the structure at the core-skin bond interfaces causes stress concentrations and delamination failures. At this point idea of using FG materials for sandwich components was suggested. In FG sandwich structures either core or skins are formed from FGMs. This ensures a continuous smooth change of material properties at the core-skin interfaces, hence concentrated stress levels are avoided in these continuously layered structures. Sandwich FG structures exhibit good strength to weight ratio, long fatigue life, excellent resistance to wear and thermal effects. These qualities make them very valuable for high-tech industries. Thus FG sandwich structures have been of great interest in recent years. In order to realize full potential of sandwich FG structures it is important to understand the mechanical behavior under various conditions. Despite this fact, minimal research has been undertaken on mechanical properties of these structures. Still there are some studies which have been conducted on buckling, static bending, free and forced vibration characteristics of FG sandwich beams. Nguyen et al. [20] suggested a higher order shear deformation beam formulation for analysis of

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buckling and free vibration of FG sandwich beams under various boundary conditions. Again Nguyen et al. [21] this time using a quasi-3D beam theory obtained analytical solutions for buckling and free vibration of FG sandwich beams. The same quasi-3D theory also used by Vo et al. [22] for static bending analysis of FG sandwich beams. In [22] finite element method (FEM) and Navier techniques were utilized for the solution of stress and deflections of FG sandwich beams under various boundary conditions and different types of cross sections. In [23] again VO et al. examine the buckling and free vibration of FG sandwich beams by FEM but this time using an improved higher order shear deformation theory. Hamilton's principle was utilized to formulate the equations of motion and boundary conditions. Yang et al. [24] used mesh free boundary domain integral equation for studying free vibration of FG sandwich beams. Based on Chebyshev collocation method Prapot et al. [25] developed mathematical models for analyzing vibration and buckling characteristics of FG sandwich beams under various boundary conditions and on two parameter elastic foundation. In order to describe the impacts of shear deformation and rotatory inertia, coupled differential equations are required to be formulated by using Timoshenko beam theory. Simsek et al. [26] worked on the free and forced vibration of FG sandwich beam under the effect of two successively moving harmonic loads with constant velocities. Assuming Timoshenko beam theory, equations of motion were obtained by Lagrange's method and solved with implicit time integration based on Newmark-β method.

Besides macroscopic applications FG sandwich beams offer advantages in the Micro-Electromechanical Systems (MEMS) and Nano-Electromechanical Systems (NEMS). They are utilized in thin film applications for Shape Memory Alloys of micro and nano scale [27], electrically excited MEM devices [28], and Atomic Force Microscopes (AFM) [29]. It is well known that size effects become important in such microscopic scales. Experiments showed that size dependent behavior is an actual phenomenon. For example it was observed that bending rigidity increases considerably as beam's thickness is reduced from 115 to 20 μm. Thus studies should be conducted on revealing the characteristics of such small structures. Classical elasticity theory does not include these small scale effects thus advanced theories are required to incorporate the size dependent behavior. Some higher order theories including size-dependent effects have been developed over recent years. In [30] Lim et al. merged the nonlocal elasticity theory of Eringen [47] and strain gradient theory of Aifantis [46] and came up with a hybrid theory which is called nonlocal strain gradient theory. This thermodynamically consistent hybrid theory was used for wave propagation analysis of Timoshenko and Euler-Bernoulli beams. Theory assumes that stress at a point is combined result of nonlocal stress [48] field and higher order gradient strain field [46]. The nonlocal stress field effect is incorporated with a nonlocal parameter (ea) and the strain gradient effect is incorporated with a length scale parameter (l_m). Simsek, in [31] applied this theory for analyzing nonlinear free vibration characteristics of FG micro-beams based on Von Kármán type geometric nonlinearity. Simsek utilized Hamilton's principle in order to obtain equations of motion and boundary conditions. Thai et al. [32] studied free vibration and buckling of FG sandwich micro-beams based on another micro-structural theory which is called modified couple stress theory. Study presented results for two main types of FG sandwich beams; one with homogenous core and FG skins, the other with homogenous skins and FG core. Li et al. developed an analytical solution by utilizing Navier method for analysis of micro Euler-Bernoulli and Timoshenko beams based on nonlocal strain gradient theory [33]. Equations of motion and boundary conditions were obtained by Hamilton's principle. In [34] Xiaobai et al. studied an axially FG beam with nonlocal strain gradient theory. Beam's material properties were varied along axial direction via power low index. Bending, buckling and free vibration equations are solved via generalized differential quadrature method. Lu et al., assuming sinusoidal shear deformation for a micro-beam, developed nonlocal strain gradient theory based Hamiltonian formulation [35]. They obtained analytical solutions via Navier technique.

Another important class of problems for MEMS development is concerning the dynamical stability of micro structures. One of the most common type of dynamic stability arises when the beam is under axially pulsating normal force which can be assumed harmonic in nature. The interaction between the frequency of the axial load and the beam's lateral vibration frequency may result in parametric instabilities and usually failure which can occur at much lower load levels than the static buckling load. Thus determining the regions of safe operation conditions so that beam is operating outside of the instability regions is utmost importance in the design of micro-beam structures under the effect of a variable axial load. This important design problem attracted few researchers in recent years. Liang et al. addressed the conditions leading to parametric instability of a FG sandwich micro-beam based on modified couple stress theory in conjunction with Timoshenko beam theory [36]. They utilized the Hamilton's principle in deriving the equations of motion and boundary conditions. In [37] Saffari et al. analyzed dynamic stability problem of a FG nano-beam which was subjected to thermal effects. They utilized Bolotin's method for the solution of the Timoshenko beam under the effect of dynamic axial loading. Again dynamic equations are derived by using Hamilton's principle. Helong [38] worked on the dynamic instability of functionally graded multilayer graphene nanoplatelet-reinforced composite (GPLRC) beams under periodic axial force and temperature change. Li [39] studied the effect of temperature on the transverse vibration of axially traveling nano-beams. The formulation was based on strain gradient elasticity theory and variational method was used to obtain the equation of motion and classical and non-classical boundary conditions. Sui et al. [40] studied the dynamic effects of axially moving FGM beam based on Timoshenko beam theory. They utilized Hamilton's principle to obtain the equation of motion and used complex mode approach to obtain the vibration modes and frequencies. Liu et al. [41] used nonlocal elasticity theory for the investigation of the transverse free vibration and stability of axially moving nano-plates. Hamilton's principle was utilized to obtain governing equations and method of separation of variables was then employed for the solution. Instability and flutter behavior were analyzed via Galerkin method. In another study Liu et al. [42] developed a formulation based on nonlocal stress gradient theory to analyze the instability problem of axially moving nano-beam with time varying velocity. Resonance frequencies were obtained after solving a sixth-order partial differential equation for different boundary conditions. They concluded that unstable regions were greatly influenced by nonlocal effects.

Besides published literature which are mentioned above, studies concerning the dynamic instability problem of sandwich micro-beams are very limited compared to the number of studies dealing with the ordinary (homogeneous) FG micro-beams. In addition, to the best of authors' knowledge, there is no reported work on the dynamic stability of sandwich FG micro-beam which is based on nonlocal strain gradient theory with various boundary conditions.

In this study, based on the nonlocal strain gradient theory (NLSGT), the dynamic stability of sandwich micro-beam is proposed in the frame work of Timoshenko beam theory with temperature effects. Using the classical rule of mixture in order to estimate the effective material properties which are changing continuously through the height of the beam, equations of motion and related boundary conditions are derived via Hamilton's principle. Differential quadrature (DQ) method is utilized to solve the obtained equations. The influences of beam's aspect ratio, material gradient index, static load factor, length scale parameter, nonlocal parameter, temperature change, various boundary conditions and different cross section shapes for sandwich configuration on the regions of dynamic instability of micro-beam are studied. Some of the present results compared with the previously published results for validation of the current proposal. Excellent agreement is observed.