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Free vibration analysis of locally damaged aerospace tapered composite structures using component-wise models

A. Viglietti, E. Zappino*, E. Carrera

MUL[2] Group, Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Italy

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ABSTRACT

This work presents the free vibration analysis of tapered aircraft structures made of composite and metallic materials, with reference to global and local damage. A refined one-dimensional model, developed in the framework of the Carrera Unified Formulation, has been used to provide a detailed description of structures. Multi-component aeronautical structures have been modeled adopting Lagrange polynomials to evaluate the displacement field over the cross-section. Each component has been described through the component-wise approach, with its own geometrical and mechanical characteristics. The effects of localized damage have been investigated, thanks to the accuracy of the layer-wise models adopted. The model has been assessed by comparing the results with classical FE models. The results show that the present approach provides an accurate solution for the free vibration analyses of complex structures and is able to predict the consequences of a global or local failure of a structural component. The computational efficiency and the accuracy of the model used in this work can be exploited to characterize the dynamic response of complex composite structures considering a large number of damage configurations.

1. Introduction

Aeronautical structures are composed of several components that distribute the loads they undergo. An increasing number of aeronautical parts are made of composite materials for weight saving purposes. It is clear that, given the multi-component nature of these kinds of structures, if one component fails, the stress distribution and the structural behavior changes according to the entity of the damage. The knowledge of these effects is a crucial point in the design process to increase the structural reliability and the safety factor. Moreover the timely damage detection of damage is important for maintenance programs. Several nondestructive tests, such as ultrasounds or the magnetic field test, already exist. However, an estimation of the location of the damage is required to increase the efficiency of these methods. The presence of the damage affects the dynamic response of a structure, and the variations in the frequencies and modal shapes can be used to detect structural damage. Several works on this kind of damage detection have been proposed. Zhang et al. [1] and Capozzucca [2] proposed analyses of damaged composite beams, studying vibration behavior. The work of Wang [3] used an FE method to detect damage in wind turbine blades considering variations of the modal shape curvatures. Nguyen [4] proposed a study on the detection of damage in which calculating the modal shapes were calculated using three-dimensional beam elements. Pollayi and Yu [5] investigated the mechanical behavior of a damaged rotor and wind turbines using beams, on the basis of the geometrically nonlinear 3-D elasticity theory and the variational asymptotic beam sectional analysis (VABS). Pérez et al. [6] adopted a different approach and performed extensive experimental analyses on the vibration of damaged laminates. The presence of damage and the characteristics of the damage can be estimated by referring to a database that includes information on the natural frequencies and model shapes of a wide spectrum of damaged cases, using accurate measurements of the real structure. This database can only be achieved through mathematical model analyses because a great deal of experimental proofs is not recommended because of time and money constraints. These models should be able to provide very accurate displacement and strain/stress fields. Damage introduces local and non-classical effects, which cannot always be detected by the conventional FE models that are used in the aeronautic field. A three-dimensional analysis is required to provide accurate results, but this can lead to huge computational costs. In this work, an advanced beam model based on the Carrera Unified Formulation is proposed to deal with damaged structures in order to obtain accurate results, but with low computational costs expressed in terms of Degrees of Freedom (DOFs). Classical theories, such as the Euler-Bernoulli beam model [7] or the Timoshenko beam model [8], are not suitable for damage detection. In the last few years, many works have

* Corresponding author.
E-mail address: enrico.zappino@polito.it (E. Zappino).

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been proposed to extend the application of one-dimensional models to any geometry, boundary condition or, mechanical complexity. In the aeronautical field, for aerodynamic reasons, particular shapes such as tapered shape or twist angle, are used. These factors increase the structural complexity and, as a result, more complex models are required. Tapered shapes are considered in this work. In this way, if the beam axis is placed in the y-axis direction, the bending stiffness $EI(y)$ changes along the axis. The classical approximation introduced to deal with such geometries is a step-by-step approach, which involves the subdivision of the structure into several rigidly prismatic beams with different cross-sections. The approximation is improved by increasing the number of subdivisions. Analytical methods, [8]9 are used to introduce the shear stress of a tapered beam. After the introduction of the FE method, several works have been proposed. A modified stiffness matrix for tapered components has been proposed by Just [10]. This work uses modified displacement functions which consider the variations in the proprieties of the sections. Brown [11] presented a stiffness matrix formulation for a linearly tapered beam, while Schreyer [12] proposed a beam theory for tapered beams, in which the shear strain is considered Many works have been proposed about aeronautical structures in the framework of the Carrera Unified Formulation. In the present 1-D CUF model, the displacement field over the cross-section is described through expansion functions. This feature allows the model to deal with arbitrary geometries, materials, and boundary conditions. After the first models, which were based on Taylor expansions, Lagrange polynomials were introduced. In this way, multi-component structures can be modeled through *ad hoc* formulations of each component (Component-Wise approach) [13]. Some of the works about this approach and its capability in the aerospace field are those of [14–16]. The work of [17] deals, through the CW approach, with different prismatic structures made of an isotropic material; several types of damage were considered. The frequencies were evaluated for each case and the modal shapes were compared using MAC (Modal Assurance Criterion)[18]. This criterion has already been employed in the civil field (damaged bridges) by Salawu and Williams [19]. The extension of the models to tapered structures has been proposed in [20,21].

In this work some aircraft structures with a tapered shape are analyzed using a 1-D CUF model, considering different types of damage. The paper is organized as follows. A first part concerns the one-dimensional model: the theory, finite element solution and model of the damage are presented. Subsequently, several results are discussed and, finally, the main remarks are presented.

2. Refined one-dimensional models formulation

The damage detection through free vibration analyses requires models with three-dimensional capabilities able to deal with complex local phenomena. Here, the Carrera Unified Formulation is presented to develop a one-dimensional refined model able to deal with this topic. After some preliminaries, the basis and the advantages of the CUF are presented in this section, finally, the damage modeling approach is introduced.

2.1. Preliminaries

At first, it's necessary to define the work space of this formulation. Two frames are used to achieve the model of a structure. The first frame (x_G,y_G,z_G) is the global coordinate system of the three-dimensional space. The beams formulation is derived at the local level, respect a second frame (x,y,z). y is the local beam axis and x,z represent the plane of the beam cross-section. The beam model derived at the local level can be arbitrary placed in the space using rotations and translations. These frames are shown in Fig. 1a.

The reference system (1,2,3) is the material reference system. The local displacement vector is expressed as:

$$\boldsymbol{u}^T(x,y,z) = \{u_x\, u_y\, u_z\} \tag{1}$$

The stress vector $\boldsymbol{\sigma}$ and the strain one ϵ are achieved as:

$$\boldsymbol{\sigma}^T(x,y,z) = \{\sigma_{xx},\sigma_{yy},\sigma_{zz},\tau_{xy},\tau_{xz},\tau_{yz}\} \tag{2}$$

$$\epsilon^T(x,y,z) = \{\epsilon_{xx},\epsilon_{yy},\epsilon_{zz},\epsilon_{xy},\epsilon_{xz},\epsilon_{yz}\} \tag{3}$$

The strain vector is defined with the following linear strain–displacement relation:

$$\epsilon = \boldsymbol{bu} \tag{4}$$

where $\boldsymbol{b}$ is a differential operator (a 6×3 matrix). The components of this matrix can be found in the book by Carrera et al. [22].

Hook's law provides the stress vector defined with the following equation:

$$\boldsymbol{\sigma} = \boldsymbol{C}\,\epsilon \tag{5}$$

where $\boldsymbol{C}$ is the 6×6 material coefficient matrix. It's a symmetric matrix, then $C_{ij} = C_{ji}$. $\boldsymbol{C}$ changes the components respect the kind of considered material. A *anisotropic* material which has a different behavior in any direction, is composed of 21 independent coefficients. Instead, if the proprieties are the same along three perpendicular planes, the material is defined as *orthotropic* material and the coefficients become nine components. In this case, the matrix $\boldsymbol{C}$ is defined as:

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{6}$$

The matrix are composed by 12 terms, but due to the symmetry of the matrix, $C_{12} = C_{21}, C_{13} = C_{31}$ and $C_{23} = C_{32}$. For this reason the matrix is reduced to 9 components. With this type of material, the preferential direction of the material should be defined. For this reason, a third reference system is introduced referred to the material. This frame is figured in 1*b*. An example of an orthotropic material is a fiber-reinforced layer. This layer lies on the plane 23 which is parallel to the plane *xy*. The axis 1 is aligned with the *z*-axis. Considering the axis 3 as the fiber direction, this one can be rotated with an angle of θ respect the *y*-axis. A positive counterclockwise rotation is considered. The present formulation allows the material to be oriented in an arbitrary direction to achieve particular lamination. As a consequence the transformation matrix $\boldsymbol{T}$ is introduced:

$$\boldsymbol{C} = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & 0 & 0 & 0 & sin2\theta \\ sin^2\,\theta & cos^2\,\theta & 0 & 0 & 0 & -\,sin2\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos\theta & -\,sin\theta & 0 \\ 0 & 0 & 0 & sin\theta & cos\theta & 0 \\ -\,cos\theta sin\theta & cos\theta sin\theta & 0 & 0 & 0 & cos^2\,\theta{-}sin^2\,\theta \end{bmatrix} \tag{7}$$

A *transformed material stiffness matrix* is introduced and it is expressed with the following form

$$\widetilde{\boldsymbol{C}} = \boldsymbol{TCT}^T \tag{8}$$

This is the new stiffness matrix to be introduced in the Hooke's law.

$$\boldsymbol{\sigma} = \widetilde{\boldsymbol{C}}\,\epsilon \tag{9}$$

If the material has the same behavior in all directions, it is a *isotropic* material. Over any direction, the material provides the same behavior. In this case, there is no need to define a material reference system and a rotation matrix. The performance of the material can be described with only one value of the Poisson ratio and of Young's modulus. These assumptions lead to have

$$C_{11} = C_{22} = C_{33} \quad C_{12} = C_{13} = C_{23} \quad C_{44} = C_{55} = C_{66} \tag{10}$$

The explicit forms of $\boldsymbol{C}$ terms can be found in the books by Tsai [23] or