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Non-linear stability and load-carrying capacity of thin-walled laminated columns in aspects of coupled buckling and coupled stiffness submatrix

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ABSTRACT

To assess a load-carrying capacity of compressed thin-walled plate structures in the paper the coupling buckling phenomenon of compressed columns was analyzed. The columns were of open cross-sections and made of coupled laminate. Selected configuration of laminate layers enables different types of coupling between membrane and bending states which describes the coupling stiffness submatrix **B**. Element values of stiffness matrix **ABD** were determined with application of classical laminate plate theory CLPT.

The main aim of the work is to estimate an influence of chosen submatrix **B** elements on buckling, post-buckling and load-carrying capacity of analyzed thin-walled structures. The problem was solved with the Koiter's theory application. The detailed computations were performed for uniformly compressed lip channel and top hat channel. The dimensions of both columns were chosen in a way which allowed to observe the strong coupling effect among different buckling modes. It were two laminate configurations considered which differed in value of stiffness reduction coefficients.

1. Introduction

Development of hydro-thermally curvature-stable laminates [1–3] caused that in a design process it is possible to analyze laminates of arbitrary layer sequences. Elements made of this type laminates can be manufactured on industrial scale with the autoclave technology. During this process pre-impregnated fiber reinforced laminas are assembled in patterns and cured in elevated temperature and pressure. This type laminates do not warp during warming and cooling. However they are prone to lay-up errors which lead to internal loads and even in extreme case to deformations. The residual loads can be observed by cut-out holes which undergo different types of deformations.

Application of laminate elements characterized by arbitrary layer arrangements allows to wider tailoring their mechanical properties but is a challenge due to coupling effects present in general laminates. This effect is based on that a membrane load can exert a curvature change whereas moments can cause in-plane deformations and vice versa [4–5]. The laminate lay-up has a crucial influence on existing types of mechanical coupling. Thus it is possible such a laminate configuration which will express chosen membrane couplings, i.e. extension-shear coupling effect [6–7], bending state couplings, i.e. bending-torsion coupling effect [8], or chosen membrane and bending state couplings, i.e. extension-twisting or bending coupling effects [9]. Type of obtained effects is imposed by the completeness of stiffness matrix **ABD** which can be determined with the classical laminate theory application [4–5]. For the coupling of membrane state are responsible elements of matrix **A**. The coupling of bending state is defined by matrix **D** elements. Different cases of coupling between membrane and bending states are an effect of nontrivial matrix **B** elements. A behaviour of arbitrary laminate is different as symmetric laminates or laminates with no mechanical coupling present.

In the literature known to the authors one can find a few works analysing the behaviour of thin-walled structures made of laminates exhibiting the effect of mechanical coupling. Vibrating and/or rotating elements [10–12] or subjected to stability loss [13–16] were investigated. The coupling of the membrane and bending states was only taken into account in [10–12] investigating vibrations of rectangular plates and in [16] investigating the phenomenon of buckling. Particularly, there is a lack of studies investigating the effect of the coupling of the membrane and bending states described by the submatrix **B,** on the behaviour of compound thin-wall structural elements during the post-buckling state. When investigating compressed plates made of gradient material when the non-trivial coupling submatrix **B** [17] occurs, the post-buckling equilibrium pathways are asymmetrical.

Works [18–20] dealt with the influence of coupling submatrix **B** on the load-carrying capacity of uniformly compressed thin-walled FML-

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FGM columns with closed [19] and open [18] cross-sections. These analyses use a multimodal approach to the description of the buckling phenomenon. The analyzed cases were characterized by strong interactions among different modes of buckling. In these works, stiffness reduction coefficients were introduced for the beam model (i.e., 1D model) in the case of FML-FGM structures. In [20] the influence of selected elements of coupling stiffness summarises on the stability and load-carrying capacity of FML-FGM of open cross-sections is presented. Only cases where the lowest buckling stress corresponded to global mode were analyzed. In this case, the interaction of the modes accelerates the phenomenon of destruction, because the value of the load-carrying capacity does not exceed the lowest buckling loads. Global buckling mode always leads to a destruction of the structure. Attention was paid to the necessity of further analysis of cases when the lowest buckling loads correspond to local mode, because it is possible that the load-carrying capacity exceeds the lowest buckling load value.

In this paper, the authors consider the influence of selected elements of coupling matrix **B** and the buckling interaction phenomenon on the load-currying capacity of composite structures for which the global buckling loads are higher than local ones. For comparison, some examples were also analyzed when there is a reverse relationship between global and local buckling loads.

2. Analytical background

The non-linear problem of stability has been solved using Koiter's theory. It is an asymptotic perturbation method [21–22]. A multi-mode buckling approach was applied to determine a load-carrying capacity [23–24]. The differential equilibrium equations of the thin-walled laminated structures were obtained with a variational method. Details can be find in paper [25]. The full Green's strain tensor are assumed:

$$\begin{aligned}\varepsilon_x &= u_{,x} + \tfrac{1}{2}(w_{,x}^2 + v_{,x}^2 + u_{,x}^2)\\ \varepsilon_y &= v_{,y} + \tfrac{1}{2}(w_{,y}^2 + v_{,y}^2 + u_{,y}^2)\\ 2\varepsilon_{xy} &= u_{,y} + v_{,x} + w_{,x}w_{,y} + u_{,x}u_{,y} + u_{,x}v_{,y}\end{aligned} \tag{1}$$

and

$$\begin{aligned}\kappa_x &= -w_{,xx}\\ \kappa_y &= -w_{,yy}\\ \kappa_{xy} &= -2w_{,xy}\end{aligned} \tag{2}$$

where: u, v, w-displacements parallel to the respective axes x, y, z of the local Cartesian system of co-ordinates, where the plane xy coincides with the middle surface of the plate before buckling.

The laminate's constitutive equations have the following form [4–5]:

$$\begin{Bmatrix}\mathbf{N}\\ \mathbf{M}\end{Bmatrix} = \begin{Bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{D}\end{Bmatrix}\begin{Bmatrix}\varepsilon\\ \kappa\end{Bmatrix} \tag{3}$$

where

$$\varepsilon = \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ 2\varepsilon_{xy}\end{Bmatrix} \kappa = \begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix} \tag{4}$$

$$\mathbf{A} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ & A_{22} & A_{26}\\ Sym. & & A_{66}\end{bmatrix}\quad \mathbf{B} = \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ & B_{22} & B_{26}\\ Sym. & & B_{66}\end{bmatrix}\quad \mathbf{D} = \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ & D_{22} & D_{26}\\ Sym. & & D_{66}\end{bmatrix} \tag{5}$$

where: the sections forces **N**, the moments **M** and the in-plane and out-plane deformations (i.e., **ε** and **κ**, respectively) and the submatrix **A** describes the composite's behaviour in its plane expresses the membrane stiffness, the submatrix **D** represents the bending stiffness and the submatrix **B** is referred to as a coupling stiffness submatrix. The classical laminate plate theory (CLPT) is employed [4–5].

The second Piola-Kirchhoff's stress tensor and the transition matrix using Godunov's orthogonalization have been applied to solve the problem. The equilibrium equations of the thin-walled laminated structures are as follows [23–25]:

$$\left(1-\frac{\sigma}{\sigma_r}\right)\zeta_r + a_{pqr}\zeta_p\zeta_q + b_{rrrr}\zeta_r^3 - \frac{\sigma}{\sigma_r}\zeta_r^* + ...=0 \quad \text{for } r = 1,...J \tag{6}$$

where: σ_r – buckling stress instead of the r-th buckling mode, ζ_r – dimensionless amplitude of the r-th buckling mode, ζ_r^* – dimensionless amplitude of the initial imperfections corresponding to the r-th buckling mode, σ – compressive stress, a_{pqr} and b_{rrrr} – constant coefficients, respectively. The range of indices: p, q, r is from 1 to J, where J is the number of interacting modes. The summation is supposed on the repeated indices. The a_{pqr} and b_{rrrr} coefficients in Eq. (6) are given for example in [23–25].

The load-carrying capacity (denoted as, σ_s) corresponding to the maximum value of the compressive stress σ for the imperfect structure than the Jacobian of the Eq. (6) is equal to zero. In the cases of a thin-walled structures with opened cross-sections an effect of the interaction between global and local buckling modes has to be taken into account. The buckling modes have to be selected to attain the lowest value of the load-carrying capacity. In the present paper, a four-mode buckling approach is applied. The flexural (S) and the flexural-torsional (A) global modes, the symmetric local (S) and the antisymmetric (A) local one are taken into consideration as in [18–19]. The symmetry conditions (i.e., S) of the buckling mode corresponds to flexural or distortional-flexural buckling, whereas the antisymmetry conditions (i.e., A) entail flexural-torsional or distortional-flexural-torsional buckling. The coupling buckling phenomenon takes place when for the analysis an arbitrary number of symmetrical modes and/or even number of antisymmetric modes will be assumed [25–26]. The employed solution method accounts also complex modes as global-distortional or distortional-local (for more detailed analysis see [18–19]). In the analysis a general layer sequence of considered composite columns was taken into account. Due to the presence of the submatrix **B**, the extensional force results not only in extensional deformations, but also in bending of the structures. The mechanical coupling affects strongly a buckling response, so the solution procedures become difficult.

The presented work includes two types of couplings, i.e. the influence of the individual components of the coupling stiffness submatrix **B** and interactive (i.e., coupled) buckling on load-carrying capacity. The main purpose of the work is to analyze how particular elements of coupling submatrix affect the buckling and load-carrying capacity of thin-walled columns for various modes of global and local buckling and their interaction.

3. Analysis of the results

For computations thin-walled prismatic columns were assumed, simply supported at both ends. All multilayered walls of the structure are flat. Each layer of laminate obeys the Hooke's law. The columns with a top hat cross-section (Fig. 1a) and a lip channel cross-section (Fig. 1b) were considered. Dimensions of the cross-sections were: $b_1 = 150$ mm; $b_2 = 50$ mm; $b_3 = 25$ mm and the thickness $t = 1.68$ mm. Each column was made of a 12-layer composite. Each layer of the thickness $t_{lay} = 0.14$ mm is characterized by the following mechanical properties [3]: Young's elastic moduli in 1, 2 material directions – $E_1 = 161.2$ GPa and $E_2 = 11.38$ GPa, shear modulus in the 1–2 plane $G_{12} = 5.17$ GPa, Poisson's ratio $\nu_{12} = 0.38$.

For the analysis it was assumed the following laminate configuration $[\theta/-\theta_2/\theta/0/\theta/-\theta/90_5]_T$, where θ is an angle of fiber reinforcement direction for particular layer measured to the compression load direction. The positive sign of angle θ corresponds to trigonometric direction [4]. For this general type of configuration some components of submatrix **A** become equal to zero $A_{16} = A_{61} = A_{26} = A_{62} = 0$ whereas all elements of submatrix **B** and **D** are different from zero (Eq.