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# Three-dimensional vibration behavior of bi-directional functionally graded curved parallelepipeds using spectral Tchebychev approach

Bekir Bediz

*Mechatronics Engineering Program, Faculty of Engineering and Natural Sciences, Sabancı University, Istanbul 34956, Turkey*

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ABSTRACT

This paper presents the application of the three-dimensional spectral-Tchebychev technique to accurately predict the vibration behavior of bi-directional functionally graded material curved parallelepipeds including geometries such as beams, thin/thick plates, and solids. In this study, the material distribution within the domain of the structure is obtained using bi-directional Mori-Tanaka method. To derive the boundary value problem governing the dynamics of functionally graded curved parallelepipeds, three-dimensional elasticity equations are used together with extended Hamilton's principle. Numerical solution of the integral boundary value problem is performed using the three-dimensional spectral Tchebychev approach. To validate and assess the performance of the presented solution approach, a number of case studies are conducted. In each case study, the non-dimensional natural frequencies and mode shapes are calculated and compared to those found using a finite element solution approach. Furthermore, computational time of the simulation is measured in each case. It is shown that the presented solution technique enables accurate prediction of vibration behavior of bi-directional functionally graded curved parallelepipeds as precise as a finite elements method, but for a fraction of the computational cost.

## 1. Introduction

Vibration behavior of functionally graded materials (FGM) is critical for a broad range of applications in diverse industries such as aerospace, automotive, and ship-building. Since its discovery in the late twentieth century, this concept has attracted increasing attention due to its flexibility to achieve desired material properties (such as to obtain high specific strength and high specific rigidity) and its wide range of applications [1–3].

FGMs are heterogeneous composite materials obtained by varying the volume composition of its constituents along selected/predefined axes [4,5]. The volume composition is generally described by a simple power law or an exponential relationship. Thus, the tailored material properties are defined as continuous functions that depend on spatial coordinates [2,6]. Compared to laminated composites where two or more materials having dissimilar material properties are bonded together, the delamination and crack initiation problems due to the sudden change of material properties (leading to undesired stress discontinuity) can be easily eliminated [2,7]. This unique characteristic of FGMs makes it one of the most promising structural materials for future novel applications in diverse fields of engineering. Therefore, it is crucial to develop a high-fidelity model that can both accurately/precisely and efficiently (in terms of computational cost) capture the dynamics of these structures as also stated in a recent review article by Swaminathan et al. [5].

In the past two decades, a large body of literature has been devoted to the modeling dynamics (vibrational behavior) of FGM structures. These studies can be mainly categorized into two groups as analytical and numerical methods. Most of the analytical studies focus on simple geometries and boundary conditions such as beam (one-dimensional – 1D) [8,9] and rectangular or circular plate (two-dimensional – 2D) models where the material properties (Young's Modulus and density) vary generally unidirectionally since it is not possible to obtain a closed form analytical solution for more complex geometries [10,5]. Furthermore, when the material is graded along more than one direction, it is highly difficult to obtain an analytical solution due to the complexity of the problem [7].

To enable the solution of more complex geometries and boundary conditions, numerical methods have been employed. For instance, beams with non-uniform cross-sections [11], bi-directional FGM beams with various (and complex) boundary conditions [12,13] can be solved. Although these beam theories yield accurate results for simple specific geometries, as the geometry becomes complex (i.e., as the aspect ratio of the beams decreases or as the vibration behavior of the structure exhibits coupled motions due to the coupling between the motions such as bending-axial (in-plane) motion or bending-twisting (out-of-plane) motion arising from the geometry/cross-section of the structure), the accuracy of these methods deteriorate. To overcome the limitations of

*E-mail address:* bbediz@sabanciuniv.edu.

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the beam-based models, many researchers benefitted from plate/shell models such as classical, Mindlin, or higher-order shear deformation plate/shell theories using Galerkin [14,15] or Ritz [16,17] methods.

In particular, shell structures are commonly employed in wide range of engineering applications due to their high-level stiffness behavior enabling efficient load carrying capabilities; therefore, there is an exponentially grown interest in understanding the dynamic behavior (i.e., predicting the natural frequencies and mode shapes) of FGM shell structures [3,18–20]. One of the main approaches used in the literature is Generalized Differential Quadrature (GDQ) method, that is developed by Shu [21], due to its simplicity and versatility. This method leads to a generalized eigenvalue problem in which the points on the middle surface of the shells are defined as the generalized displacements [22–25]. To take into account the curvature effect, if necessary, a curvilinear coordinate system is introduced and the derivative and integral operations are performed using this coordinate system [3,16,26,27]. Furthermore, in a recent study of Tornabene et al. [28], the vibrational behavior of arbitrarily shaped doubly-curved laminated composite shell structures are presented where a mapping procedure based on the use of Non-Uniform Rational B-Splines is performed and the integral form of the partial differential equations is solved using Generalized Integral Quadrature method. Note that in these technique, the derivative and integral operations are performed numerically; thus, the precision of the solution technique highly depends on the sampling scheme and the selected basis functions [3,29].

Another common numerical approach to model the dynamics of the FGM structures is to use finite element (FE) methods (both for 1D, 2D, and three-dimensional – 3D- problems) [30,31]. Although FE approach enables accurate prediction of the dynamic behavior of FGMs, the modeling procedure (finding a suitable mesh, defining the varying material properties across the elements in the mesh, etc.) is arduous and more importantly imposes a significant computational burden to obtain a converged solution. However, there is a huge demand for 3D analysis of FGM structures that can accurately capture the dynamics of these structure and reduce the computational burden simultaneously [5].

Alternatively, to increase the computational efficiency, series-based solution approaches such as Rayleigh–Ritz [32] or Galerkin's [33] methods are employed. Furthermore, the computationally efficient nature of the series based approaches are combined with the generality of the finite elements approach using methods such as spectral element method (SEM) and quadrature element method (QEM) [34,35]. Although these techniques are computationally efficient, their drawbacks are **(i)** the difficulty in selecting proper basis/trial functions that needs to be determined for each different geometry and boundary conditions, **(ii)** the necessity to use special numerical algorithms.

Recently, a novel series based approach has been developed by the author to predict the (coupled) 3D dynamics of complex engineering structures including stationary and rotating parallelepipeds, pretwisted and curved structures having isotropic material properties [29,36–38]. This technique uses 3D elasticity equations to derive the integral boundary value problem (IBVP) through the extended Hamilton's principle. Compared to the techniques in the literature, it has many advantages. First of all, IBVP approach incorporates the natural boundary conditions directly to the problem and simplifies the formulation. Thus, derivation of the partial differential form of the BVP is eliminated. Furthermore, the projection matrices approach is used to impose the essential boundary conditions on the IBVP; thereby applying different basis/trial functions for each different structure and boundary conditions is eliminated. Finally, the IBVP is discretized using Tchebychev polynomials (that has exponential convergence [39,40]) and the system matrices are calculated through the exact evaluation of differentiation and inner-product operations using the Tchebychev matrix operators and Galerkin's method [41]. Therefore, compared to the collocation methods [42,43] where Tchebychev polynomials or any other computationally efficient polynomials are used, the presented approach in this study necessitates that the integrals of the equations vanish with respect to all polynomials of a certain degree instead of only at sampling points [29].

This paper presents the application of the 3D spectral Tchebychev (3D-ST) technique for solving the 3D (coupled) dynamics of bi-directional FGM curved parallelepipeds under various different boundary conditions. The integral boundary value problem (IBVP) is derived using the extended Hamilton's principle where the strain energy of the FGM structure is obtained using 3D elasticity equations. To facilitate the spatially varying material properties along one or more directions, the Mori-Tanaka scheme is utilized. Then, if necessary, a coordinate transformation is defined to map the curved geometry into a simple parallelepiped geometry to simplify the domain of the problem. Following the simplification of the domain of the problem, the spectral-Tchebychev approach is benefitted to discretize the derived IBVP. To validate the accuracy/precision of the presented approach and demonstrate its capabilities and computational performance, a number of case studies (including straight and curved parallelepipeds having uniform, uni-axially or bi-axially varying material properties) are investigated. The results obtained through the presented solution approach and the duration (CPU –central processing unit– time) of the simulations are compared to those found from a commercial finite elements (FE) software.

## 2. Derivation of the model

The generic geometry of a curved parallelepiped is depicted in Fig. 1. Here, instead of the global ($x\,y\,z$) cartesian coordinate frame, a curvilinear coordinate frame ($\bar{x}\,\bar{y}\,\bar{z}$ coordinate frame) placed at the geometric center of the parallelepiped, is used to describe the geometry: $L_x$ and $L_y$ are the lengths of the parallelepiped along $\bar{x}$ and $\bar{y}$ axes, and $h$ is the height (or thickness) of the parallelepiped. $R$ denotes the radius of curvature.

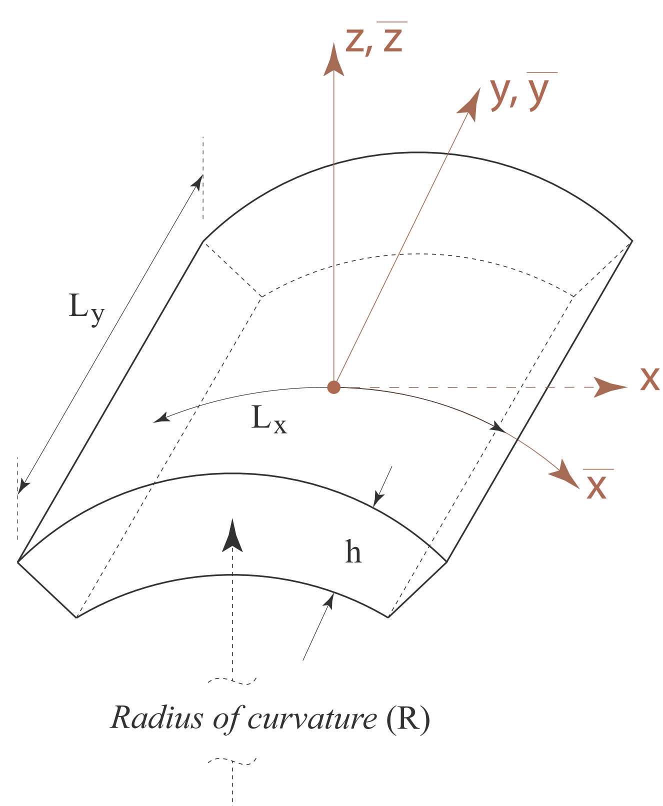


**Fig. 1.** Generic description of the geometry of a curved parallelepiped.