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Letter to the Editor

Inappropriateness in composite laminate orthotropic elasticity formulations and numerical implementation of laminate elasto-plasticity within backward Euler integration algorithm

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ABSTRACT

Since contradictions occur among literatures in the mathematical formulation of composite lamina's orthotropic elasticity and one doubt exists in one published paper's numerical procedure about the iteration convergence during the Newton iteration for plastic strains based on backward Euler algorithm, the incorrect formulation of orthotropic elasticity is necessary to be modified and the convexity of yielding function respect to plastic consistency factor is recommended to be declared to guarantee the convergence of Newton iteration where the iterative results are more preferable based on converged state variables. This letter aims at modifying the published incorrect orthotropic elasticity model and perfecting the proposed implementation procedures of predicting composite laminate plastic deformation with the backward Euler algorithm and Newton iteration.

1. Introduction

With the widespread applications of thermoplastic polymer matrix composite laminates, precise engineering analysis techniques of those composite laminate's elasticity, nonlinear plasticity as well as diverse damage forms, are in demand for tailoring laminate structures with superior load capacity and impact resistance than that of conservative designed ones.

During past decades, amount of literatures have sprung up in describing the elasticity, nonlinear plasticity as well as damage behaviors of laminates. Prior to nonlinear behaviors, orthotropic elasticity were utilized by researchers [1–8] and predicted linear responses proved satisfying. However, contradictions occur in the modeling of lamina's orthotropic elasticity among those published articles [1–8], which may make engineering analysts and researchers confused and even result in misunderstandings.

In terms of laminate nonlinear plasticity, Liao and Liu [5]constructed an elasto-plastic model with isotropic hardening and implemented Newton iteration procedure based on backward Euler integration algorithm into subroutines. This approach was effective in dealing with lamina's isotropic hardening plasticity, including projecting trial stresses onto yielding surfaces and updating local plastic deformation. Although this numerical procedure obtained satisfactory predictions of plasticity compared with experimental results, the non-declaration of yielding function's convexity and the iteration based on non-converged state variables' values are questionable in terms of convergence of the local iteration.

With above considerations, the incorrect and correct formulations of lamina's orthotropic elasticity are necessary to be declared, and the more proper numerical procedure about solving lamina's plasticity with isotropic hardening based on backward Euler algorithm and Newton iteration has to be elaborated on.

2. Discussions

2.1. The mathematical formulation of lamina's orthotropic elasticity

Unidirectional laminas being the unit ply components in stacked polymer-based composite laminates could be taken as orthotropic materials. The three-dimensional orthotropic elasticity is formulated as [8].

$$\boldsymbol{\varepsilon} = \boldsymbol{S}\bullet\boldsymbol{\sigma} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix}\bullet\boldsymbol{\sigma} \tag{1}$$

where E_1, E_2, E_3 are respectively young's modulus along longitudinal direction and in-plane and out-of-plane transverse directions. G_{12}, G_{23}, G_{31} are shear moduli and $\nu_{ij}(i \neq j$ and $i,j = 1,2,3)$ are Poisson ratios. Therefore, the elastic stiffness matrix $\boldsymbol{C}$ could be obtained via the inverse of the

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compliance matrix $\boldsymbol{S}$.However, two different representations of orthotropic elastic stiffness matrix $\boldsymbol{C}$ occurs in literatures. One formulation given in Refs. [1,5–7] is as Eq. (2).

$$\boldsymbol{C}=\frac{1}{\Omega}\begin{bmatrix} E_1(1-\nu_{23}\nu_{32}) & E_2(\nu_{12}-\nu_{13}\nu_{32}) & E_3(\nu_{13}-\nu_{12}\nu_{23}) & 0 & 0 & 0\\ E_1(\nu_{21}-\nu_{31}\nu_{23}) & E_2(1-\nu_{13}\nu_{31}) & E_3(\nu_{23}-\nu_{21}\nu_{13}) & 0 & 0 & 0\\ E_1(\nu_{31}-\nu_{21}\nu_{32}) & E_2(\nu_{32}-\nu_{12}\nu_{31}) & E_3(1-\nu_{12}\nu_{21}) & 0 & 0 & 0\\ 0 & 0 & 0 & \Omega G_{12} & 0 & 0\\ 0 & 0 & 0 & 0 & \Omega G_{23} & 0\\ 0 & 0 & 0 & 0 & 0 & \Omega G_{31} \end{bmatrix} \tag{2}$$

in which $\Omega=1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{13}\nu_{31}-2\nu_{12}\nu_{31}\nu_{23}$. The other formulation in Refs. [2–4,8] is as Eq. (3).

$$\boldsymbol{C}=\frac{1}{\Omega}\begin{bmatrix} E_1(1-\nu_{23}\nu_{32}) & E_2(\nu_{12}+\nu_{13}\nu_{32}) & E_3(\nu_{13}+\nu_{12}\nu_{23}) & 0 & 0 & 0\\ E_1(\nu_{21}+\nu_{31}\nu_{23}) & E_2(1-\nu_{13}\nu_{31}) & E_3(\nu_{23}+\nu_{21}\nu_{13}) & 0 & 0 & 0\\ E_1(\nu_{31}+\nu_{21}\nu_{32}) & E_2(\nu_{32}+\nu_{12}\nu_{31}) & E_3(1-\nu_{12}\nu_{21}) & 0 & 0 & 0\\ 0 & 0 & 0 & \Omega G_{12} & 0 & 0\\ 0 & 0 & 0 & 0 & \Omega G_{23} & 0\\ 0 & 0 & 0 & 0 & 0 & \Omega G_{31} \end{bmatrix} \tag{3}$$

Theoretically, $\boldsymbol{C}\cdot\boldsymbol{S}=\boldsymbol{I}$ should always hold and Eq. (3) is in accordance with this requirement, whereas Eq. (2) does not meet the condition. Therefore, the formulation Eq. (2) adopted in articles [1,5–7] was incorrect to represent composite lamina's orthotropic elasticity and was necessary to be modified into Eq. (3) to avoid further misunderstanding.

2.2. Prediction of laminate plasticity via Newton iteration based on backward Euler algorithm

2.2.1. Typical solution procedure for laminate plasticity

Dealing with composite laminate nonlinear plasticity, backward Euler integration algorithm combined with Newton iteration is typically an effective approach [5]. Fig. 1 presents the schematic explanation of the solution procedure, which can be concluded as follows:

a. The trail elastic status variables $(\bullet)_{n+1}^{\mathrm{trial}}$ which initially freezes the current plastic increments are firstly introduced, where $\bullet$ could denote elastic strain $\boldsymbol{\varepsilon}^{\mathrm{e}}$, plastic strain $\boldsymbol{\varepsilon}^{\mathrm{p}}$, equivalent plastic strain $\widetilde{\varepsilon}^{\mathrm{p}}$ and elastic stress $\boldsymbol{\sigma}^{\mathrm{e}}$.
b. The yielding criterion $F(\boldsymbol{\sigma}_{n+1}^{\mathrm{e,trial}},\widetilde{\varepsilon}_{n+1}^{\mathrm{p,trial}})\leqslant 0$ is checked and the Kuhn-Tucker loading/unloading conditions [9] has to be satisfied.
c. When no yielding occurs, the trail status is kept and no further plastic deformation happens in current increment step.
d. When yielding occurs, local Newton iteration with backward Euler integration algorithm is utilized to project trail stresses onto the yielding surface to obey the Kuhn-Tucker conditions, in which plastic consistency factor denoted as $\Delta\lambda$ in paper [5] is obtained for update the status variables $(\bullet)_{n+1}$.

2.2.2. Declaration of yield function's convexity for the convergence of Newton iteration result

As illustrated in Fig. 1, noticeably, when the local Newton iteration is conducted to search for feasible plastic consistency parameter $\Delta\lambda$ in the yielding step, only the convexity of the yielding function with respect to plastic consistency parameter is ensured could the convergence of the searched plastic consistency factor be guaranteed [9]. Then, the local plastic increment could be obtained.

However, typically seen in Ref. [5], the convexity of the yielding function with respect to plastic consistency parameter was not proved, which was necessary to be declared with the consideration of mathematical rigorousness. If the convexity is not guaranteed, the non-converged plastic factor will make adverse impact on the discrepancies between predicted plastic deformation and experimental results.

2.2.3. The numerical iteration procedure for updating plastic deformation

The Newton iteration procedure based on backward Euler integration algorithm for determining plastic deformation denoted as Eq. (25) in Ref. [5] is formulated as:

$$\begin{aligned} &\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p},k+1}=\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p},k}+\delta\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p},k+1}\\ &\widetilde{\varepsilon}_{n+1}^{\mathrm{p},k+1}=\widetilde{\varepsilon}_{n+1}^{\mathrm{p},k}+\delta\widetilde{\varepsilon}_{n+1}^{\mathrm{p},k+1}\\ &\boldsymbol{\sigma}_{n+1}^{\mathrm{e},k+1}=\boldsymbol{C}:(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p},k+1})\\ &\Delta\lambda_{n+1}^{k+1}=\Delta\lambda_{n+1}^{k}-\frac{F(\Delta\lambda_{n+1}^{k})}{\partial F(\Delta\lambda_{n+1}^{k})/\partial\Delta\lambda_{n+1}^{k}} \end{aligned} \tag{4}$$

where the fictitious $(k+1)$th trail status variables $(\bullet)_{n+1}^{k+1}$ were all calculated by the corresponding non-converged variables $(\bullet)_{n+1}^{k}$ in the kth iteration.

Although it is mathematically reasonable, two deficiencies exist in this handle [9]. When the decrease of the plastic consistency factor occurs in the iteration based on the instant non-converged values, new iterations are necessary to iterate from the initial converged values. Moreover, it is not satisfying to cope with path-dependent plasticity.

Consequently, a recommended iteration procedure which iterates based on the converged values $(\bullet)_n$ in the nth step at the initialization time t_{n} is elaborated on as