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Wave propagation characteristics in a piezoelectric coupled laminated composite cylindrical shell by considering transverse shear effects and rotary inertia

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ABSTRACT

Wave behavior is investigated in a piezoelectric coupled laminated fiber-reinforced composite cylindrical shell by considering the transverse shear effects and rotary inertia. A mathematical model is presented for analysis of wave propagation in a laminated fiber-reinforced composite cylindrical shell coated with the piezoelectric layer. By solving an eigenvalue problem, dispersion characteristics for different wave modes are obtained. Piezoelectric effect on dispersion curve is investigated. The effects of material properties of host substrate shell, laminate stacking sequence, and fiber orientation on dispersion curve are examined as well. In addition, a comparison of dispersion solutions from different shell theories with different axial and circumferential wave numbers and piezoelectric layer thickness is provided to illustrate the transverse shear and rotary inertia effects on wave behavior of a laminated fiber-reinforced composite shell. The results of this paper can be used for studies on wave propagation in piezoelectric coupled laminated composite shell structures and in design of smart composite shell structures with the piezoelectric material for health monitoring and dynamic stability evaluation.

1. Introduction

Composite shell structures are widely used in aerospace, civil, marine, offshore, and other engineering industries, due to their high stiffness and strength to weight ratios compared to traditional materials. Composite materials consist of two or more constituents, whose mechanical properties and performance are superior of those of the constituent materials acting independently. One of the constituents is usually discontinuous, stiffer, and stronger and is called the reinforcement fiber, while the less stiff, weaker, and lightweight constituent is continuous and is called the matrix [1]. A composite laminate may be made up of unidirectional fiber-reinforced Carbon/Epoxy and E-glass/Epoxy layers stacked together in a specified sequence. The composite structure, then is lightweight, stiff, and in many aspects with excellent mechanical performances and properties. However, such structures may be low resistant to dynamic loading and working conditions. Understanding dynamic behavior and wave propagation in laminated composite shell structures has great importance due to high rate of frequency and dynamic strain subjected to these structures. To characterize the dynamic behavior of shell structures, dispersion solution and frequency determination have great importance. By wave propagation analysis in composite shell structures, existence of any fault, such as crack and delamination, can be detected. Therefore, understanding wave propagation in laminated fiber-reinforced composite shell structures coupled with the piezoelectric layer is very important in design of laminated composite cylindrical shell structures and smart materials required for the stability analysis and structural health monitoring.

Different shell theories were developed in past decades, which can help to model and understand dynamics of composite shell structures. The lowest-order shell theory, i.e. membrane shell theory, was developed by Love [2], in which transverse or out-of-plane shear forces (V_{xz} and $V_{\theta z}$), bending and twisting moments (M_{xx}, $M_{\theta\theta}$, and $M_{x\theta}$) are assumed negligible. Such model is usable and efficient to study thin shell structures in which only the in-plane normal and shear forces (N_{xx}, $N_{\theta\theta}$, and $N_{x\theta}$) applying in the mid-surface of shell are considered. This lower-order shell model presents the essential features of the shell and is used as a fundamental model for higher-order shell theories. Some notable works based on this simply model were presented by Donnel [3], Flugger [4], Vlasov [5], and Sanders [6].

The classical shell theory, proposed by Love [2] and Reissner [7] as the first approximation to thin shell theory, is based on the following assumption: (a) the laminate is thin compared to its lateral dimension; (b) the deflection of shell is small; (c) straight lines normal to the

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middle surface remain straight and normal to that surface after deformation; and (d) the transverse shear stresses (τ_{xz}, $\tau_{\theta z}$) are zero. Usually, the model developed based on the above assumptions is referred as the Love's bending shell theory or the classical (lower-order) shell theory. Many studies were performed on shell structures based on the Membrane and Love's bending shell theories. However, the above assumptions are not valid for thicker shells and shells with low stiffness central plies undergoing significant transverse shear deformation. Mirsky and Hermmann [8] considered the shear effects in both axial and circumferential directions and the rotary inertia effects for cylindrical shells with moderate thickness. Lin and Morgan [9] developed equations for axillary symmetric motions considering shear effects and rotary inertia. Cooper and Naghdi [10] developed a theory including both transverse shear effects and rotary inertia for non-axillary symmetric motion of shell structures. Greenspon [11] showed that the Cooper-Naghdi shell theory [10], considering both transverse shear effect and rotary inertia, is sufficient for wave propagation analysis in thicker cylindrical shells. The Cooper-Naghdi shell theory is also known as the first-order shear deformation shell theory. Based on the Cooper-Naghdi shell theory, the assumption of normality of straight lines is removed, that is, straight lines normal to the middle surface remain straight but not normal to that surface after deformation.

Last decades, the applications of smart materials and structures such as piezoelectric coupled beam, plate, and shell have been studied widely. Piezoelectric actuators are widely used in sensing and damage detection of various structures such as beams, plates and shells due to their actuating properties and unique sensing. Interdigital transducer (IDT) is used to apply wave propagation in piezoelectric structures for structural health monitoring. To this purpose, a piezoelectric layer is bonded on the surface of structures. IDT is used to excite a wave propagation in the piezoelectric coupled structure for the analysis of wave signal for the purpose of damage detection [12]. However, a proper model of wave propagation in the piezoelectric coupled shell structure is required for health monitoring. Different mechanical models have been proposed for piezoelectric actuators and sensors coupled with various structures such as beams, plates, and shells [12–19]. Vibration and wave propagation in pure piezoelectric structures have been studied previously by Mindlin [20] Tiersten [21] and Bleustein [22]. Wave propagation and vibrational analyses in cylindrical shell structures have also been studied in many research works in past decades [23–27]. However, the wave propagation in piezoelectric coupled cylindrical shells has been studied only in the last decade. Wang [12,17,18] studied theoretically the wave propagation in piezoelectric coupled metallic cylindrical shells by the membrane, Love bending, and Cooper-Naghdi shell theories. Dong and Wang [19] provided wave propagation characteristics in the piezoelectric coupled cylindrical shell with the effects of large deformation and rotary inertia, however, they did not consider the effects of the transverse shear, and only the in-plane stresses were considered in their study and transverse shears stresses and the resultant shear forces are assumed negligible. In another study by Dai and Zheng [28], the buckling behavior of a laminated cylindrical shell of functionally graded material (FGM) with the piezoelectric fiber-reinforced composite (PFRC) actuators were analyzed in thermal environment without considering the transverse shear effects and rotary inertia in mathematical modeling. Nasihatgozar et al. [29] investigated the buckling response of piezoelectric composite panels reinforced with carbon nanotubes using the classical laminated plate theory (CLPT) in which the transverse shear effects and rotary inertia were not included. Since higher wave numbers are usually required for wave propagation studies, shear deformable theories are necessary in the mathematical modeling of wave propagation in the piezoelectric coupled laminated composite shell structures. However, to the authors' knowledge, there is no theoretical study on investigating the effects of piezoelectricity, fiber orientation, and stacking sequence on wave propagation of a laminated fiber-reinforced composite cylindrical shell coupled with the piezoelectric layer by considering the effects of both transverse shear and rotary inertia via the Cooper-Naghdi shell theory. The main objective of this study is to fill this void by solving this wave propagation problem by including the transverse shear effects and rotary inertia, i.e. the Cooper-Naghdi shell theory [10].

In this paper, we investigate wave propagation in a piezoelectric coupled laminated fiber-reinforced composite cylindrical shell using the Cooper-Naghdi shell theory in which the transverse shear effects and rotary inertia are included. A complete mathematical model of wave propagation solution in a piezoelectric coupled laminated fiber-reinforced composite cylindrical shell is provided. By solving an eigenvalue problem, the phase velocity and dispersion curve for different axial wave numbers and wave modes will also be provided. The validity of this solution method and numerical results are proved with the results of the Ref. [18] by considering aluminium as core material of the piezoelectric coupled shell. Furthermore, the effects of circumferential wave number, piezoelectric layer thickness, laminate stacking sequence, core materials, and fiber orientation on dispersion solutions are examined. A comparison of dispersion solutions by different shell theories is provided as well. The results of this study can lead to a better understanding of dynamics of the piezoelectric coupled laminated composite shell structures.

2. Methodology and modeling

Configuration of a laminated fiber-reinforced composite cylinder and an infinitely long laminated fiber-reinforced composite cylindrical shell coated with a piezoelectric layer are shown in Fig. 1a and b. Coordinate x represents the direction along the shell axial direction, θ for the circumferential direction, and z for the radial direction (Fig. 1b). The $x-\theta$ plane is equidistant from the top and bottom surfaces of the laminate and is called the reference plane or mid-plane. The stress and moment resultants at an infinitesimal element for a shell structure is shown in Fig. 1c. The general strain-displacement relations in the cylindrical coordinate system (x,θ,z) are given by,

$$\varepsilon_{xx}=\frac{\partial u}{\partial x},\quad \varepsilon_{\theta\theta}=\frac{1}{R}\left(\frac{\partial v}{\partial\theta}+w\right),\quad \varepsilon_{zz}=\frac{\partial w}{\partial z},$$
$$\gamma_{x\theta}=\frac{\partial v}{\partial x}+\frac{\partial u}{R\partial\theta},\quad \gamma_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z},\quad \gamma_{\theta z}=\frac{1}{R}\frac{\partial w}{\partial\theta}+\frac{\partial v}{\partial z}-\frac{v}{R},\tag{1}$$

where u, v, and w are the displacements of a generic point of cylinder in x, θ, and z directions, respectively, and R is the reference plane radius.

2.1. Constitutive equations for a laminated composite cylindrical shell by the Cooper-Naghdi shell theory

Cooper and Naghdi [10] proposed a shell theory, i.e. the Cooper-Naghdi shell theory, in which the transverse shear and rotary inertia had been included for modeling of shell structures. Constitutive equations based on the Cooper-Naghdi shell theory for a laminated composite cylindrical shell are derived in this section. Fig. 2 shows a section of a laminated composite shell normal to the θ-axis before and after deformation, including the effects of transverse shear, where straight lines normal to the middle surface remain straight but not normal to that surface after deformation. The result of this deformation is the rotation of the cross-section $ABCD$ by angle α_x to a location $\acute{A}\acute{B}\acute{C}\acute{D}$, which is normal to the deformed middle surface [1,30]. The displacements of a generic point B based on the Cooper-Naghdi shell theory are expressed as [30],

$$u(x,\theta,z,t)=u_0(x,\theta,t)+z\alpha_x(x,\theta,t),\tag{2a}$$

$$v(x,\theta,z,t)=v_0(x,\theta,t)+z\alpha_\theta(x,\theta,t),\tag{2b}$$

$$w(x,\theta,z,t)=w_0(x,\theta,t),\tag{2c}$$

where u, v, and w are the displacements of a generic point in x, θ, and z directions, respectively and u_0, v_0, and w_0 are the reference plane displacements in x, θ, and z directions, respectively, and α_x and α_θ are the