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Vibration of a rotating composite beam with an attached point mass

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ABSTRACT

The free vibration of a rotating laminated composite beam with an attached point mass is investigated. The Ritz method with algebraic polynomials is used in the formulation. The boundary conditions are considered as clamped-free. Different shear deformation theories (first order and third order) and classical beam theories are used in the formulation. Cross-ply lamination configurations are considered. Effects of the ratio of attached mass to the beam mass, rotation speed, hub ratio, orthotropy ratio, position of attached mass, beam theory and length to thickness ratio are analyzed in detail. Some typical mode shapes are presented in order to illustrate the effects of the attached mass.

1. Introduction

Composite materials are preferred in mechanical and aerospace engineering applications (such as turbine wing, helicopter rotor blade, manipulator etc.) due to their low weight ratio and high strength. In various systems a mass is added into the system in order to increase performance. The attached point mass in rotating composites structures can be used, for example, to regulate the airflow in the wind turbine or to increase the flexibility in the car's fan and to change the vibration frequencies in the helicopter rotor.

Some studies can be obtained related to vibration of rotating beams with attached mass. Hoa [1] investigated vibration of rotating beams with attached mass using finite element method. Boyce et al.[2] obtained vibration frequencies of rotating uniform bars with a tip mass at a constant speed. Jones [3] investigated frequencies of rotating beams with attached mass using the integral equation method for different boundary conditions. Hyun [4] examined transverse vibration of a rotating beam carrying tip mass with the integral equation method. Lee [5] investigated the effects of tip mass on vibration of rotating beam with a mass attached at free end. Bhat [6] studied mode shapes and natural frequencies of a uniform cantilever beam via Ritz method. Some results are given with several parameters such as root radius, the speed of rotation, tip mass and the moment of inertia of tip mass. A dynamic model of the rotating beam system with a tip mass was obtained using a finite element model considering viscous damper and drag force by Hui et al.[7]. Park and Kim [8] studied the dynamic characteristic of a curved beam attached tip mass by adding the effects of Coriolis and centrifugal force. Song and Librescu [9] have studied the vibrational and piezo characteristics of a thin-walled beam with a rotating tip mass. Craig [10] investigated the vibration of a rotating beam with a concentrated tip mass. The vibration of a beam is studied with delamination along the width and the mass at the end point by the Jafari [11]. The frequency analysis of a rotating beam with a mass at its end is studied by Ansari [12], taking into account the torsional-bending vibrations. The coupled bending-bending vibration of an elastically constrained and mass-added beam was investigated by Lin et al. [13]. Khulief [14] studied the vibration of a tapered beam with end mass using the finite element method. Kwok et al. [15] modeled with a blade modeled in their study. In this model, the vibration of the blade attached mass for arbitrary angle is examined.

The vibration of rotating composite beams is a very complicated problem in general, if one considers pre-twist of the composite beam and coupling of flapwise-lagwise and axial vibrations. The effect of the Coriolis force, and coupled vibration of pre-twisted rotating composite beams has been investigated in a general manner in some of the previous studies [16,17]. Vibration analysis of rotating thin-walled composite box beams can be obtained in [18–20].

Free vibration of composite thin-walled rotating beams with arbitrary closed sections has been investigated by Song and Librescu [19]. Jung et al. [20] used a mixed approach in order to study the dynamics of rotating and nonrotating composite beams and blades with general geometry (open and closed cross section). Chandramani et al. [18] studied vibration of higher-order-shearable pretwisted rotating composite blades using the Galerkin method.

In a recent work, Rafiee et al. [21]. have been investigated rotating nanocomposite thin-walled beams undergoing large deformation. In this study, a general computational model has been developed to study the nonlinear steady state static response and free vibration of thin-

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walled carbon nanotubes/fiber/polymer laminated multiscale composite beams and blades. Galerkin method has been used in the formulation of the problem.

Recently, Rafiee M et al. [22] reviewed the dynamics of rotating composite beams and blades. They have included the effect of added mass on the vibration of rotating composite beams with added mass. Effects of tip mass addition on the dynamics of rotating composite beams and blades has been considered in [23–25]. According to the authors' best knowledge vibration of rotating composite beams with attached mass has been limited to only tip mass in a few studies.

The purpose of the present study is to investigate the mass effect (attached to any point of the composite beam) on the flapwise vibration of rotating composite beams. Ritz method is used in the solution of the problem. The displacement field is assumed in algebraic polynomial form. Effects of rotation speed, hub ratio, orthotropy ratio, mass ratio, position of mass and beam theories on the frequencies are investigated in detail.

2. Analysis

Consider a laminated composite beam which is rotating about a fixed axis. Dimensions of the beam are length L, with b and height h (Fig. 1).

The composite beam is assumed to be constructed of arbitrary number, N, of linearly elastic transversely isotropic layers. Therefore, the stress state in each layer is given by the generalized Hooke Law as follows

$$\sigma_x^{(k)} = Q_{11}^{(k)}\varepsilon_x \tag{1}$$

$$\tau_{xz}^{(k)} = Q_{55}^{(k)}\gamma_{xz} \tag{2}$$

where $Q_{ij}^{(k)}$ are the well-known reduced stiffness and k is the number of layers. Assuming that the deformations of the beam take place on the x-z plane and upon denoting the displacement components along the x, y and z directions by U, V and W respectively, the following displacement field can be written in the framework of generalized shear deformation beam theory [26–29]

$$U^{(k)}(x,y,z;t) = u^{(k)}(x,y;t) - zw_{,x} + \Phi^{(k)}u_1^{(k)}(x,y;t) \tag{3}$$

$$V^{(k)}(x,y,z;t) = 0 \tag{4}$$

$$W^{(k)}(x,y,z;t) = w(x,y;t) \tag{5}$$

where u_1 is the shear deformation at mid-plane. The displacement model yields the following linear kinematic relations:

$$\varepsilon_x = u_{,x} - zw_{,xx} + \Phi(z)u_{1,x} \tag{6}$$

$$\gamma_{xz} = \Phi' u_1 \tag{7}$$

where a prime denotes the derivative with respect to z and "$,x$" represent partial derivative with respect to x. Following beam theories are used in the present study.

EBT (Euler-Bernoulli theory): $\Phi(z) = 0$ (8)

FSDT (First shear deformation theory): $\Phi(z) = z$ (9)

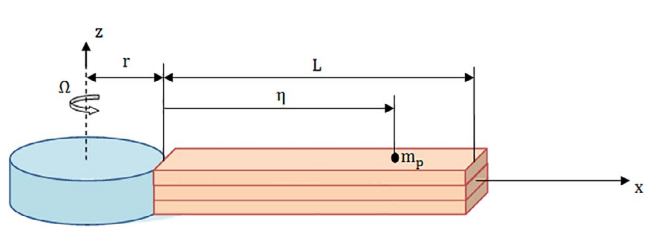

Fig. 1. Configuration of rotating composite beam with an attached point mass.

PSDBT (Parabolic shear deformation theory): $\Phi(z) = z(1-4z^2/3h^2)$ (10)

The force moment resultants can be defined as:

$$N_x^c, M_x^c = \int_{-h/2}^{h/2} \sigma_x(1,z)dz \tag{11}$$

$$Q_x^a = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz}\Phi'(z)dz \tag{12}$$

$$M_x^a = \int_{-h/2}^{h/2} \sigma_x\Phi(z)dz \tag{13}$$

By substituting the stress-strain relations into the expressions of the force and moment resultants of the present theory the following constitutive equations are obtained (20)–(23):

$$\begin{bmatrix} N_x^c \\ M_x^c \\ M_x^a \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ & D_{11} & F_{11} \\ Sym & & H_{11} \end{bmatrix} \begin{bmatrix} u_{,x} \\ -w_{,xx} \\ u_{1,x} \end{bmatrix} \tag{14}$$

$$[Q_x^a] = [A_{55}][u_1] \tag{15}$$

The extensional, coupling, bending and transverse shear rigidities are defined as follows:

$$A_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}dz \tag{16a}$$

$$A_{55} = \int_{-h/2}^{h/2} Q_{55}^{(\chi)}(\Phi)^2dz \tag{16b}$$

$$B_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}zdz \tag{16c}$$

$$E_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}\Phi(z)dz \tag{16d}$$

$$D_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}z^2dz \tag{16e}$$

$$F_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}\Phi(z)zdz \tag{16f}$$

$$H_{11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)}(\Phi)^2dz \tag{16g}$$

where $()' = d()/dz$. In these definitions, the resultants denoted with a superscript 'c' are the conventional ones of the classical beam theories; whereas the remaining ones with a superscript 's' are additional quantities incorporating the transverse shear deformation effects.

Total strain energy consists of three parts as given below:

$$U_{total} = U_{str} + U_{rot} + U_{mass} \tag{17}$$

U_{str} denotes elastic strain energy, U_{rot} is the energy due to rotation of the composite beam and $U_{masscent}$ is the energy due to attached mass. These energies are

$$U_{str} = \frac{1}{2}\int_0^b\int_0^L \{A_{11}u_{,x}^2 - 2B_{11}w_{,xx}u_{,x} + 2E_{11}u_{1,x}u_{,x} + D_{11}w_{,xx}^2 - 2F_{11}u_{1,x}w_{,xx} + H_{11}u_{1,x}^2 + A_{55}u_1^2\}dxdy \tag{18}$$

$$U_{rot} = \frac{1}{2}\int_0^L T(x)w_{,x}^2dx \tag{19}$$

where T(x) is the centrifugal force due to rotation of composite beam and defined as: