Composite Structures 189 (2018) 127–136

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Free vibration analysis of two directional functionally graded beams using a third order shear deformation theory

Armağan Karamanlı*

Department of Mechatronics, Faculty of Engineering and Architecture, Istanbul Gelişim University, Istanbul, Turkey

ARTICLE INFO

Keywords:
Two directional functionally beam
Free vibration analysis
Third order shear deformation theory
Lagrange equations

ABSTRACT

This paper presents the free vibration behavior of two directional functionally graded beams subjected to various sets of boundary conditions which are simply supported (SS), clamped-simply supported (CS), clamped-clamped (CC) and clamped-free (CF) by employing a third order shear deformation theory. The material properties of the beam vary exponentially in both directions. In order to investigate the free vibration response, the equations of motion are derived by means of Lagrange equations. The axial, transverse deflections and rotation of the cross sections are expressed in polynomial forms including auxiliary functions which are used to satisfy the boundary conditions. The verification and convergence studies are performed by using computed results from a previous study which is based on the Timoshenko beam formulation. The results for extensive studies are provided to understand the influences of the different gradient indexes, various aspect ratios and boundary conditions on the free vibration responses of the two directional functionally graded beams.

1. Introduction

Functionally Graded Materials (FGMs) are a class of composites that have received great attention in many modern engineering applications such as military, aerospace, nuclear energy, biomedical, automotive, civil engineering and marine. Due to its lower transverse shear stresses, high resistance to temperature shocks and no interface problems through the layer interfaces, the researchers have extensively examined the static, vibration and buckling responses of these structures during the last decade [1–24]. However, the conventional FGMs (or 1D-FGM) with material properties which vary in one direction are not efficient to satisfy the technical requirements such as the temperature and stress distributions in different directions for aerospace craft and shuttles [25].

The mentioned deficiency of the conventional FGM can be eliminated by using a new type FGM with material properties varying in desired directions. The mechanical and thermal behaviors of two-directional FG structures have been investigated so far. The Element Free Galerkin Method is employed to optimize the natural frequencies of 2D two-directional functionally graded beams (FGBs) in [26]. The static and thermal deformations of bi-directional FGBS are investigated by employing the state-space based differential quadrature method obtain the semi-analytical elasticity solutions [27]. A symplectic elasticity solution for static and free vibration analyses of 2D-FGBs with the material properties varying exponentially in [28]. The fully coupled thermo-mechanical behavior of 2D-FGBs is studied using an isogeometric finite element model in [29]. Free and forced vibration of Timoshenko 2D-FGBS under the action of a moving load is investigated in [30]. The buckling of Timoshenko beams composed of 2D-FGM is studied in [31]. The static behavior of the 2D-FGBs by using various beam theories is presented in [32]. An analytical solution for the static deformations of the bi-directional functionally graded thick circular beams is developed based on a new shear deformation theory with a logarithmic function in the postulated expression for the circumferential displacement in [33]. The flexure behavior of the two directional FG sandwich beams by using a quasi-3D theory and the SSPH (Symmetric Smoothed Particle Hydrodynamics) method is studied in [34].

As it is seen from above discussions, most of the studies are related to the static, dynamic and buckling analysis of conventional functionally graded (1D-FG) beams. The studies related to two directional FGBs are still limited. As far as author aware, there is no reported work on the free vibration analysis of the two directional FBGs based on a third order shear deformation theory. Main differences of this paper from the related paper [30] are: the present theory does not require a shear correction factor which depends on the material and geometrical properties as well as boundary conditions [35] of the 2D-FGBs and satisfies the zero traction boundary condition of the top and bottom surfaces of the beam, the second and third natural frequencies of the 2D-FGBs for various end conditions, aspect ratios and gradient indexes are presented within this paper and it is clear that the accuracy of the

* Address: Istanbul Gelişim University, 34215 Istanbul, Turkey.
E-mail address: afkaramanli@gelisim.edu.tr.

https://doi.org/10.1016/j.compstruct.2018.01.060
Received 24 August 2017; Received in revised form 1 December 2017; Accepted 19 January 2018
0263-8223/

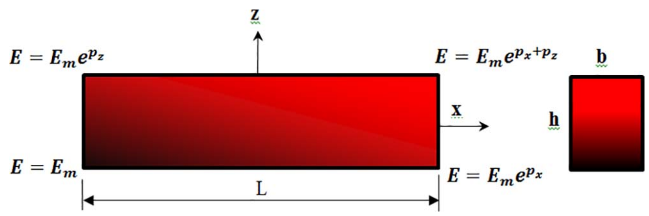

Fig. 1. Geometry and coordinate of a two-directional FGB.

Table 1
Kinematic boundary conditions used for the numerical computations.

BC	$x = -L/2$	$x = L/2$
S-S	$u = 0, w = 0$	$w = 0$
C-S	$u = 0, w = 0, \phi = 0,\ w' = 0$	$w = 0$
C-C	$u = 0, w = 0, \phi = 0,\ w' = 0$	$u = 0, w = 0, \phi = 0,\ w' = 0$
C-F	$u = 0, w = 0, \phi = 0,\ w' = 0$	

Table 2
Boundary exponents for various boundary conditions.

BC	Left end			Right end		
	p_u	p_w	p_ϕ	q_u	q_w	q_ϕ
SS	1	1	0	0	1	0
CS	1	2	1	0	1	0
CC	1	2	1	1	2	1
CF	1	2	1	0	0	0

Timoshenko beam theory decreases as the mode number increases [7]. As a result, a third order shear deformation theory is necessary to have a better prediction of vibration responses of the two directional FGBs. The main novelty of this paper is that the free vibration behavior of the two directional FGBs is analyzed based on a third order shear deformation theory by using the Lagrange equations with four different end conditions for the first time.

2. Theory and formulation

2.1. Homogenization of material properties

A two-directional functionally graded beam of length L, width b and thickness h is shown in Fig. 1. The material properties of the beam vary exponentially not only in the z-direction (thickness direction) but also in the x-direction (along the length of the beam). The Young's modulus E, shear modulus G, Poissons's ratio ν and mass density ρ vary according to the following expressions [30]

$$E(x,z) = E_m e^{p_x\left(\frac{x}{L}+\frac{1}{2}\right)+p_z\left(\frac{z}{h}+\frac{1}{2}\right)}$$
$$G(x,z) = \frac{E(x,z)}{2(1+\nu_m)}$$
$$\rho(x,z) = \rho_m e^{p_x\left(\frac{x}{L}+\frac{1}{2}\right)+p_z\left(\frac{z}{h}+\frac{1}{2}\right)} \tag{1}$$

where E_m,ν_m and ρ_m are the material properties of the reference material value at the point $(-L/2,\ -h/2)$, p_x and p_z are the gradient indexes which determine the material properties through the thickness (h) and length of the beam (L), respectively. When the p_x and p_z are set to zero then the beam becomes homogeneous.

2.2. Kinetic, strain and stress relations

The following displacement field is given for the third order shear deformation theory (Reddy Beam Theory (RBT))

$$U(x,z,t) = u(x,t) + z\phi(x,t) - \alpha z^3\left(\phi(x,t) + \frac{\partial w(x,t)}{\partial x}\right)$$
$$W(x,z,t) = w(x,t) \tag{2}$$

Here u and w are the axial and transverse displacements of any point on the neutral axis ,ϕ is the rotation of the cross sections, $\alpha = 4/(3h^2)$. By using the Eq. (2), the strain-displacement relations of the RBT are given by

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} - \alpha z^3\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)$$
$$\gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = \phi + \frac{\partial w}{\partial x} - \beta z^2\left(\phi + \frac{\partial w}{\partial x}\right) \tag{3}$$

Table 3
Verification and convergence studies, dimensionless fundamental frequencies (λ_1) of SS two directional FGBs with respect to gradient index and aspect ratio change.

Beam theory		p_x	L/h = 5 p_z						L/h = 20 p_z					
			0	0.2	0.4	0.6	0.8	1	0	0.2	0.4	0.6	0.8	1
Timoshenko [30]		0	2.6767	2.6748	2.6669	2.6533	2.6337	2.6103	2.8369	2.8349	2.8251	2.8115	2.7919	2.7685
RBT	2 terms		2.9433	2.9402	2.9310	2.9157	2.8947	2.8682	3.1468	3.1436	3.1342	3.1187	3.0972	3.0700
	4 terms		2.6780	2.6753	2.6672	2.6539	2.6354	2.6121	2.8380	2.8351	2.8267	2.8127	2.7933	2.7689
	6 terms		2.6773	2.6746	2.6665	2.6532	2.6347	2.6114	2.8371	2.8343	2.8258	2.8118	2.7925	2.7681
	8 terms		2.6773	2.6746	2.6665	2.6532	2.6347	2.6114	2.8371	2.8343	2.8258	2.8118	2.7925	2.7681
	10 terms		2.6773	2.6746	2.6665	2.6532	2.6347	2.6114	2.8371	2.8343	2.8258	2.8118	2.7925	2.7681
	12 terms		2.6773	2.6746	2.6665	2.6532	2.6347	2.6114	2.8371	2.8343	2.8258	2.8118	2.7925	2.7681
Timoshenko [30]		0.4	2.6728	2.6689	2.6611	2.6474	2.6279	2.6044	2.8330	2.8291	2.8212	2.8076	2.7880	2.7626
RBT	2 terms		2.9448	2.9417	2.9325	2.9172	2.8961	2.8695	3.1525	3.1493	3.1399	3.1243	3.1027	3.0755
	4 terms		2.6740	2.6740	2.6713	2.6632	2.6497	2.6312	2.8350	2.8322	2.8237	2.8097	2.7904	2.7660
	6 terms		2.6722	2.6694	2.6613	2.6479	2.6293	2.6059	2.8326	2.8298	2.8213	2.8073	2.7880	2.7636
	8 terms		2.6722	2.6694	2.6613	2.6479	2.6293	2.6059	2.8326	2.8298	2.8213	2.8073	2.7880	2.7636
	10 terms		2.6722	2.6694	2.6613	2.6479	2.6293	2.6059	2.8326	2.8298	2.8213	2.8073	2.7880	2.7636
	12 terms		2.6722	2.6694	2.6613	2.6479	2.6293	2.6059	2.8326	2.8298	2.8213	2.8073	2.7880	2.7636
Timoshenko [30]		1	2.6455	2.6416	2.6337	2.6201	2.6005	2.5771	2.8095	2.8056	2.7978	2.7841	2.7646	2.7412
RBT	2 terms		2.9522	2.9491	2.9398	2.9245	2.9033	2.8766	3.1820	3.1788	3.1693	3.1536	3.1318	3.1044
	4 terms		2.6527	2.6500	2.6418	2.6283	2.6096	2.5860	2.8193	2.8165	2.8080	2.7941	2.7749	2.7505
	6 terms		2.6452	2.6425	2.6343	2.6208	2.6022	2.5788	2.8089	2.8061	2.7977	2.7839	2.7647	2.7405
	8 terms		2.6452	2.6425	2.6343	2.6208	2.6022	2.5788	2.8089	2.8061	2.7977	2.7839	2.7647	2.7405
	10 terms		2.6452	2.6425	2.6343	2.6208	2.6022	2.5788	2.8089	2.8061	2.7977	2.7839	2.7647	2.7405
	12 terms		2.6452	2.6425	2.6343	2.6208	2.6022	2.5788	2.8089	2.8061	2.7977	2.7839	2.7647	2.7405