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# Dynamics properties of composite sandwich open circular cylindrical shells

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ABSTRACT

This paper deals with the dynamic properties of three-layered composite sandwich open circular cylindrical shells (CSOCCS). First, the equations of motion that govern the free vibrations of CSOCCS are derived by applying Hamilton's principle based on the first-order shear deformation shell theory. Owing to considering the effect of rotary inertias and shear deformation, thin-to-moderately thick shells can be analyzed. Next, these equations are solved by means of the closed-form Navier method. The calculated results are compared with the findings of previous studies and those obtained by the finite element method, and a good agreement is observed. The variation of modal loss factor and frequency with system parameters is evaluated and presented graphically. It is the first time to study the dynamic properties of composite open circular cylindrical shells with constrained viscoelastic core.

## 1. Introduction

A typical sandwich structure consists of two stiff face layers that carry the great portion of the bending load separated by a light inner core that has energy dissipating property [1]. Composite sandwich structures are very effective in sound insulation and reducing vibration response of lightweight and flexible structures, where the soft core is strongly deformed in shear, due to the adjacent stiff layers [2]. CSOCCS is a special form of closed circular cylindrical sandwich shells, and often used as structural components of pressure vessels, roof structures, open space buildings, and marine structures.

A huge amount of research efforts have been devoted to the vibration analysis of sandwich structures, such as beams, plates and shells. Since 1958 and the work of Kerwin and Edwar [3], later, Ross et al. [4], DiTaranto [5], and Mead and Markus [6] extended Kerwin's work, and various kinds of shell theories have been proposed and developed. Among them, thin shell theories neglecting the shear deformation [7–12]; first order shear deformation theories [13,14]; and recently, the high-order shear deformation theory [15,16] widely applied to dynamic behaviors, optimal design, bending, impacting and vibration. And, by making some different assumptions and simplifications, various sub-series thin shell theories were developed, such as Goldenveizer–Novozhilov's theory [7,8], Reissner–Naghdi's theory [9,10], and Donnel–Mushtari's theory [11,12], etc, some review articles described these theories in detail [17–19].

On the basis of these theories, many relevant studies of open circular cylindrical shells have been carried out, quite extensively, by both analytical and numerical methods in the subsequent decades. Suzuki and Leissa [20] developed an exact solution procedure for determining the free vibration frequencies and mode shapes of open cylindrical shells. Yu et al. [21] obtained the exact solutions using the generalized Navier method for the free vibration analysis of open circular cylindrical shells with different combinations of boundary conditions based on the Donnell–Mushtari's theory. Selmane and Lakis [22] presented a method for the dynamic and static analysis of thin, elastic, anisotropic and non-uniform open cylindrical shells. Lim et al. [23] presented a three-dimensional elastic analysis of the vibration of open cylindrical shells, and obtained the natural frequencies and vibration mode shapes via a three-dimensional displacement-based extremum energy principle. Zhang and Xiang [24] developed an analytical procedure for determining the free vibration frequencies of open circular cylindrical shells with intermediate ring supports based on the Flügge thin shell theory. Kandasamy and Singh presented different numerical methods based on the Rayleigh-Ritz method for the forced vibration of open cylindrical shells [25] and the free vibration of skewed open circular cylindrical deep shells [26]. L.K. Abbas et al. [27] used the transfer

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matrix method to analyze natural frequencies and mode shapes of open-variable thickness circular cylindrical shells in a high-temperature field, and employed the fourth-order Runge-Kutta method to solve the matrix equation. S. Joniak et al. [28] proposed an analytical formula of critical stresses, and investigated the elastic buckling and limit load of open circular cylindrical thin shells in pure bending state with the use of the finite element method. Ye T. et al. [29,30] presented a unified formulation to investigate the vibrations of composite laminated deep open shells with various shell curvatures and arbitrary restraints, including cylindrical, conical and spherical ones. Z. Su et al. [31] studied the free vibration analysis of moderately thick functionally graded open shells with general boundary conditions with Rayleigh-Ritz method. X.L. Yao et al. [32] employed an analytical solution of the traveling wave form to study the free vibration analysis of open circular cylindrical shells based on the Donnell-Mushtari-Vlasov thin shell theory. Punera and Kant [33] presented the free vibration analysis of functionally graded open cylindrical shells by using various refined higher order theories. Q. Wang et al. [34] presented a new three-dimensional exact solution for free vibration of thick open cylindrical shells on Pasternak foundation with general boundary conditions.

Through the correlative literature reviews, we can see that most of the existing works of the laminated composite open circular cylindrical shells (LCOCCS) were restricted to the undamped open cylindrical shells, and concretely concentrated on the following aspects: free and force vibration, solution methods, dynamic analysis, bucking and bending, and so on. To the authors' knowledge, there is no work reported on the dynamic properties of CSOCCS. However, in practice, CSOCCS have been widely applied to the field of engineering and technology; therefore, it becomes very important and necessary to study the dynamics properties of CSOCCS, thus providing some useful numerical results for the practical engineering applications of CSOCCS.

## 2. Vibration equation of CSOCCS

The geometry and dimension of the CSOCCS with total thickness $h$ and subtended angle $\theta_0$ is shown in Fig. 1, and axial (x), circumferential (y) and radial (z) coordinate system are defined.

The symbols $R_i$ and $h_i$ (i = 1, 2, 3) denote radius and thickness of the ith layer, and the subscripts 1, 2, and 3 in the following derivation are designated for base shell, viscoelastic core and constraining layer, respectively. The length of open cylindrical shells is $a$. In the base layers

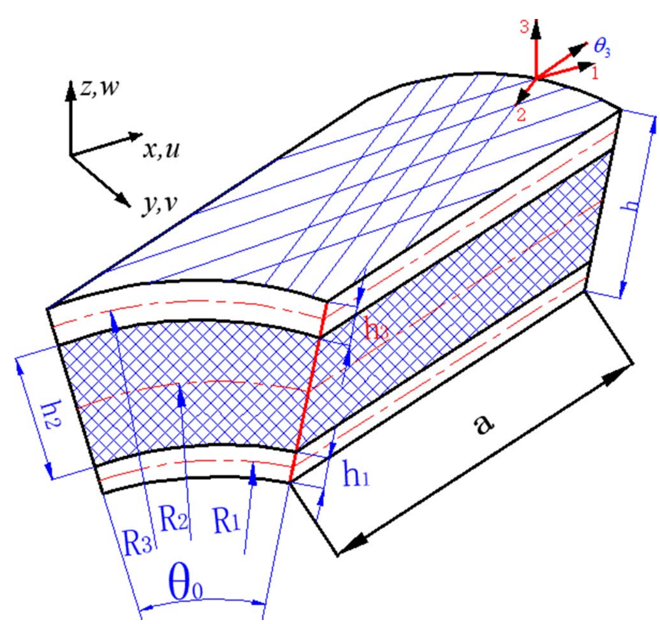


**Fig. 1.** Geometry and coordinate system of the CSOCCS.

and constraining layers, 1, 2, 3 represent the principal coordinate system of the composite material. 1 and 2 represent the direction parallel to and perpendicular to fiber orientation respectively. And 3 denote the direction perpendicular to 1–2 plane. The angle between the principal coordinate of the composite material in *ith* layer and the *x*-axis of the shell is named angle of fiber and denoted by $\theta_i$.

In order to derive the governing equations, assumptions are as follows: 1) shear modulus and elasticity modulus of the viscoelastic material adopt the constant complex modulus, 2) normal strain is negligible, 3) transverse displacement of all points on a normal to the shell is constant, 4) there is no slip between the layers.

### 2.1. Kinematic relations

Based on the first-order shear deformation shell theory and the above assumptions, the displacements in the middle surface of each layer are expressed by:

$$\begin{aligned}
\overline{U_i}(x,\theta,z,t) &= u_i(x,\theta,t) + z^{(i)}\alpha_i(x,\theta,t)\\
\overline{V_i}(x,\theta,z,t) &= v_i(x,\theta,t) + z^{(i)}\beta_i(x,\theta,t)\\
\overline{W_i}(x,\theta,z,t) &= w(x,\theta,t)
\end{aligned} \tag{1}$$

where, $i = 1,2,3$. $t$ is time variable; $\overline{U_i}$, $\overline{V_i}$ and $\overline{W_i}$ are the generalized displacements of ith layer along the axial, circumferential and radial coordinates, respectively. $u_i$,$v_i$ are the axial and circumferential displacements of the middle surface of *ith* layer, respectively. $\alpha_i$,$\beta_i$ are the rotations of transverse normal to center plane with respect to the circumferential and axial coordinate, respectively. The $z^{(i)}$ is measured with respect to the mid-plane of the *ith* layer, $-h_i/2 \le z^{(i)} \le h_i/2$, $i = 1,2,3$.

The linear strain–displacement relations of *ith* layer space are written as follows:

$$\begin{aligned}
\varepsilon_{xx}^{(i)} &= \partial u_i/\partial x + z^{(i)}\partial\alpha_i/\partial x\\
\varepsilon_{\theta\theta}^{(i)} &= \partial v_i/R_i\partial\theta + w/R_i + z^{(i)}\partial\beta_i/R_i\partial\theta\\
\gamma_{x\theta}^{(i)} &= \partial u_i/R_i\partial\theta + \partial v_i/\partial x + z^{(i)}(\partial\alpha_i/R_i\partial\theta + \partial\beta_i/\partial x)\\
\gamma_{\theta z}^{(i)} &= k_c(\partial w/R_i\partial\theta + \beta_i - v_i/R_i)\\
\gamma_{xz}^{(i)} &= k_c(\partial w/\partial x + \alpha_i)
\end{aligned} \tag{2}$$

where, $k_c$ is the shear correction factor. $\varepsilon_{xx}^{(i)}$, $\varepsilon_{\theta\theta}^{(i)}$, $\gamma_{x\theta}^{(i)}$ represent *ith* normal strain components in the middle surface of the shell along coordinate axis; $\gamma_{\theta z}^{(i)}$, $\gamma_{xz}^{(i)}$ represent *ith* shear strain components in the middle surface along coordinate axis.

According to the generalized Hooke's law, the stress–strain relation of the *ith* layer can be expressed, as follows:

$$\begin{pmatrix}\sigma_{11}^{(i)}\\ \sigma_{22}^{(i)}\\ \sigma_{12}^{(i)}\\ \tau_{23}^{(i)}\\ \tau_{13}^{(i)}\end{pmatrix} = \begin{pmatrix} Q_{11}^{(i)} & Q_{12}^{(i)} & 0 & 0 & 0\\ Q_{12}^{(i)} & Q_{22}^{(i)} & 0 & 0 & 0\\ 0 & 0 & Q_{66}^{(i)} & 0 & 0\\ 0 & 0 & 0 & Q_{44}^{(i)} & 0\\ 0 & 0 & 0 & 0 & Q_{55}^{(i)}\end{pmatrix}\begin{pmatrix}\varepsilon_{11}^{(i)}\\ \varepsilon_{22}^{(i)}\\ \gamma_{12}^{(i)}\\ \gamma_{23}^{(i)}\\ \gamma_{13}^{(i)}\end{pmatrix} \tag{3}$$

where, $\sigma_{11}^{(i)}$,$\sigma_{22}^{(i)}$,$\sigma_{12}^{(i)}$ represent *ith* normal stress components; $\tau_{23}^{(i)}$,$\tau_{13}^{(i)}$ represent *ith* shear stress components. The reduced stiffness components $Q_{mn}^{(i)}$ of *ith* layer are given, as follows,

$$\begin{aligned}
Q_{11}^{(i)} &= \frac{E_1^{(i)}}{1-\nu_{12}^{(i)}\nu_{21}^{(i)}},Q_{12}^{(i)} = \frac{\nu_{12}^{(i)}E_2^{(i)}}{1-\nu_{12}^{(i)}\nu_{21}^{(i)}},Q_{22}^{(i)} = \frac{E_2^{(i)}}{1-\nu_{12}^{(i)}\nu_{21}^{(i)}},Q_{44}^{(i)} = G_{23}^{(i)},Q_{55}^{(i)}\\
&= G_{13}^{(i)},Q_{66}^{(i)} = G_{12}^{(i)},i = 1,2,3
\end{aligned}$$

Among them, $E_1^{(i)}$, $E_2^{(i)}$, $G_{12}^{(i)}$, $G_{13}^{(i)}$, $G_{23}^{(i)}$, $\nu_{12}^{(i)}$, $\nu_{21}^{(i)}$ represent elasticity modulus, shear modulus, and Poisson's ratio of *ith* layer, respectively.

By coordinate transformation, the stress–strain relations in the coordinate system can be obtained as