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Mechanical behavior of damaged laminated composites plates and shells: Higher-order Shear Deformation Theories

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ABSTRACT

The paper aims to present a novel mathematical formulation for the modelling of damage. In particular, the decay of the mechanical properties of the elastic media is modeled by means of two-dimensional smooth functions, which are the Gaussian and the ellipse shaped ones. Various damaged configurations are obtained as concentrated variations of the elastic properties of the materials by setting properly the parameters that define the distributions at issue. This approach is employed to investigate the dynamic behavior of damaged plates and shells made of composite materials. In particular, a massive set of parametric studies is presented for this purpose. The results are obtained numerically by means of the Generalized Differential Quadrature (GDQ) method and are presented in terms of natural frequencies. Several Higher-order Shear Deformation Theories (HSDTs), which can include also the Murakami's function to capture the so-called zig-zag effect, are used and compared.

1. Introduction

Composite materials are currently employed in many engineering and physics fields. Their analysis and development, in fact, involve several aspects related to chemistry, material science, production technology, and solid mechanics [1]. Their fast growth is due to the need of an improved and more efficient structural response [2]. In general, this requirement can be fulfilled by combining together two (or more) constituents [3].

Let us consider for instance a fiber-reinforced medium, which is known as a typical composite. This peculiar material is composed by long reinforcing fibers, which provide the strength and stiffness features, embedded in a matrix material, which keeps together the reinforcing phase and allows the stress transfer among adjacent fibers [3]. Thus, its mechanical behavior turns out to be superior to the one that can be reached by a conventional material, such as an isotropic medium. Fiber-reinforced media represent the basic constituents of laminated composite materials. In general, these layers (or plies) are assembled together to achieve the value of strength and stiffness required by a certain application. It should be recalled that each fiber-reinforced layer can be studied as an orthotropic material, whose orientation can be arbitrarily chosen. In other words, the orientation of the fiber-reinforced layers and their geometric arrangement are the fundamental parameters that have to be considered to properly design a laminate [4]. Thus, several lamination schemes (or stacking sequence) can be obtained for this purpose [5–21]. For the sake of completeness, it should be recalled that various mathematical approaches and homogenization techniques are developed to esteem the mechanical properties of fiber-reinforced composites, as shown in the recent paper by Tornabene et al. [22].

As far as the structural mechanics is concerned, fiber-reinforced layers, as well as laminated composites, are widely used as main constituents of shells, plates, and beams [23]. These structural elements made of innovative materials are employed in many engineering fields, such as mechanical, civil, automotive, biomedical, and aerospace, due to the advantages that the composite media can exhibit [24]. These advantages are even more evident when curved structures, such as shells, are analyzed. Their peculiar shapes, in fact, define an outstanding efficiency in bearing external forces. In addition, higher values of strength and stiffness can be reached without increasing the structural weight [24]. The geometry of doubly-curved shells is well-defined once their middle surfaces are specified. For this purpose, an extremely accurate and rigorous description of these curved geometries can be performed by means of the analytical expressions provided by the differential geometry, as illustrated in the book by Kraus [25]. This approach allows to overcome those difficulties related to the description of a two-dimensional surface characterized by arbitrary radii of curvature [26–31].

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Nevertheless, these composite materials and advanced constituents could experience a failure mechanism in their service life. This condition could be caused by matrix cracks, fiber-matrix debonds, fiber fractures, fiber pull-out, and delaminations, as clearly illustrated in the book by Reddy and Miravete [32]. These mechanism are described accurately in the works by Highsmith and Reifsnider [33], Dumont et al. [34], Ladevèze and Le Dantec [35], Daudeville and Ladevèze [36], Tay and Lim [37], Lambros and Rosakis [38], Muc and Kędziora [39], Zhang and Zhang [40], Abisset et al. [41], Batra et al. [42], and Murakami [43]. Further analyses concerning related topics can be found in the following list of recent papers [44–56].

In general, the term damage is introduced to specify a decay of the mechanical properties of the composite. As a consequence, the structural behavior is modified by the onset of these phenomena. As in the previous research by Tornabene et al. [57], the deterioration of the mechanical properties is modeled through the application of peculiar two-dimensional smooth functions, which are the Gaussian and the ellipse shaped ones, within the reference domain. These mathematical laws allow to define a concentrated variation of the elastic properties at issue, if the parameters that define these distributions are properly set.

The main aim of the current paper is to investigate the effect of the damage on the dynamic response of laminated composite plate and shell structures. For this purpose, a set of parametric studies is presented to analyze the variation of the natural frequencies of these structures as a function of the point of application, intensity, and width of the damage. Thus, the analyses are carried out regardless of the causes that could determine a specific damage and the consequent decay of the elastic properties.

The mechanical behavior of the laminated composite shell structures in hand is investigated by means of a higher-order formulation. In particular, the governing equations are obtained according to the theoretical framework provided by a Unified Formulation (UF) [58–66], since these peculiar mechanical configurations could require a Higher-order Shear Deformation Theory (HSDT) to be analyzed correctly. The formulation at issue is able also to capture the effect of a soft-core by means of the well-known Murakami's function, which can be embedded in the kinematic model [67–71]. This approach is generally included in the Equivalent Single Layer (ESL) theory, since all the mechanical parameters, as well as the degrees of freedom of the problem, are computed in the shell middle surface [72–90].

The solution of the dynamic problem at issue is achieved numerically by means of the Generalized Differential Quadrature (GDQ) method, due to its accuracy and stability features [91–97]. To this aim, both the theoretical formulation and the numerical tool are implemented in a MATLAB code developed by the authors [98].

2. Shell geometry

The description of the shell geometry is achieved accurately through the principles of differential geometry [25]. To the best of the authors' knowledge, this approach represents in fact an analytical tool that allows to describe efficiently a curved surface. Thus, it turns out to be extremely useful to overcome the difficulties related to the definition of those doubly-curved surfaces that are taken as middle surfaces of shell structures.

A global reference system $Ox_1x_2x_3$ must be introduced, as well as the corresponding unit vectors $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$, as shown in Fig. 1. Let us consider a generic doubly-curved shell element of constant thickness h. Its middle surface represents the reference domain for the governing equations. Thus, the local reference system $O'\ \alpha_1\ \alpha_2\ \zeta$ should be conveniently introduced within this surface. It should be pointed out that α_1,α_2 are orthogonal and principal coordinates, whereas ζ denotes the direction orthogonal to the middle surface itself. The position vector $\mathbf{r}(\alpha_1,\alpha_2)$ is required to identify each point P' of the shell middle surface. It should be recalled that the position vector $\mathbf{r}(\alpha_1,\alpha_2)$ assumes a different meaning according to the shell that has to be analyzed. Once this vector is specified, it is possible to identify also any point P of the three-dimensional medium by means of the corresponding position vector $\mathbf{R}(\alpha_1,\alpha_2,\zeta)$ defined below

$$\mathbf{R}(\alpha_1,\alpha_2,\zeta) = \mathbf{r}(\alpha_1,\alpha_2) + \zeta\ \mathbf{n}(\alpha_1,\alpha_2) \tag{1}$$

where $\mathbf{n}(\alpha_1,\alpha_2)$ stands for the outward unit normal vector, which can be evaluated by means of the following cross product (denoted by $\times$)

$$\mathbf{n} = \frac{\mathbf{r}_{,1}\times\mathbf{r}_{,2}}{|\mathbf{r}_{,1}\times\mathbf{r}_{,2}|} \tag{2}$$

in which $\mathbf{r}_{,i} = \partial\mathbf{r}/\partial\alpha_i$. The Lamè parameters $A_1(\alpha_1,\alpha_2)$, $A_2(\alpha_1,\alpha_2)$ of the surface at issue are obtained by means of a scalar product (denoted by $\cdot$)

$$A_1 = \sqrt{\mathbf{r}_{,1}\cdot\mathbf{r}_{,1}},\quad A_2 = \sqrt{\mathbf{r}_{,2}\cdot\mathbf{r}_{,2}} \tag{3}$$

The doubly-curved surfaces used in this paper are characterized by principal radii of curvature $R_1(\alpha_1,\alpha_2)$, $R_2(\alpha_1,\alpha_2)$ that vary in each point of the domain. These quantities can be computed as follows

$$R_1 = -\frac{\mathbf{r}_{,1}\cdot\mathbf{r}_{,1}}{\mathbf{r}_{,11}\cdot\mathbf{n}},\quad R_2 = -\frac{\mathbf{r}_{,2}\cdot\mathbf{r}_{,2}}{\mathbf{r}_{,22}\cdot\mathbf{n}} \tag{4}$$

where $\mathbf{r}_{,ii} = \partial^2\mathbf{r}/\partial\alpha_i^2$. Finally, the three-dimensional size effect linked to the shell curvature is taken into account by means of the geometric parameters $H_1(\alpha_1,\alpha_2,\zeta)$, $H_2(\alpha_1,\alpha_2,\zeta)$ defined below

$$H_1 = 1 + \frac{\zeta}{R_1},\quad H_2 = 1 + \frac{\zeta}{R_2} \tag{5}$$

The principal coordinates α_1,α_2 define a finite domain by setting the following limitations

$$\alpha_1^0 \leqslant \alpha_1 \leqslant \alpha_1^1,\quad \alpha_2^0 \leqslant \alpha_2 \leqslant \alpha_2^1 \tag{6}$$

where α_i^0,α_i^1, for $i = 1,2$, stand for the minimum and the maximum values, respectively. On the other hand, the third coordinate ζ is included within the thickness of the shell as shown below

$$-\frac{h}{2} \leqslant \zeta \leqslant \frac{h}{2} \tag{7}$$

It should be recalled that the overall thickness of the structure h is given by

$$h = \sum_{k=1}^{l} h_k \tag{8}$$

if a laminated composite shell is analyzed. The total number of layers is denoted by l, whereas k identifies the geometric and mechanical parameters of the k-th ply. Thus, h_k stands for the thickness of the k-th layer, which is computed as follows

$$h_k = \zeta_{k+1} - \zeta_k \tag{9}$$

where ζ_k and ζ_{k+1} are respectively the lower and upper coordinates of the layer, as shown in Fig. 1.

3. Shell structural model

An enriched kinematic model is used in this paper to define the displacement field of a generic laminated composite shell. Since an ESL approach is developed, each mechanical quantity is defined on the reference surface. The maximum order of kinematic expansion N is chosen arbitrarily to obtain several shear deformation theories.

3.1. Displacement field

The three-dimensional displacements $U_1(\alpha_1,\alpha_2,\zeta,t)$, $U_2(\alpha_1,\alpha_2,\zeta,t)$, $U_3(\alpha_1,\alpha_2,\zeta,t)$ are given by

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