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Axial dispersion in weakly turbulent flows of yield stress fluids

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ABSTRACT

We analyze turbulent flows of shear-thinning yield stress fluids in both pipe and channel geometries. We lay down a consistent procedure for hydraulic calculation of Herschel-Bulkley fluids; i.e. finding the relationship between the mean velocity and the wall shear stress. We show that for weakly turbulent flows it is necessary to include an analysis of wall layers in studying dispersion. In pipe flows, we observe an $\mathcal{O}(10)$ increase in Taylor dispersion coefficients, compared to highly turbulent values. This arises from a combination of large velocity and small turbulent dispersivity, acting over a wall layer that can represent $\gtrsim 20\%$ of the pipe area. In channel flows the wall layer effect is more modest, but still represents an $\mathcal{O}(1)$ increase in Taylor dispersion coefficient. The preceding effects are negated for small power law index, due to rapid reduction of the wall layer, and are observed to reduce modestly as the yield stress increases.

1. Introduction

The aim of this paper is to explore the effects of the yield stress on dispersion of mass in weakly turbulent duct flows. The motivation comes from studying the dispersive flows that are found in the primary cementing of oil and gas wells. During primary cementing a sequence of different fluids are successively pumped into the well, travelling downwards within the casing (pipe) and returning upwards along the outside of the casing (narrow eccentric annulus); see [42]. The initial stages of wells are vertical. Within the past 10–20 years the industrial trend has been towards wells that are longer and frequently drilled horizontally. Extended reach drilling leads to larger frictional pressure drops and horizontal wells mean that frictional pressure is more important in relation to violating pore-frac pressure bounds. Together, these have meant that modern wells are less frequently cemented in highly turbulent flow regimes. Laminar, transitional and weakly turbulent flow regimes are more usual.

The fluids used in primary cementing are drilling fluids, washes, spacer fluids and cement slurries, all of which are characterised within the industry as shear-thinning yield stress fluids, e.g. Herschel-Bulkley fluids. If water-based, these fluids are miscible. In turbulent flows they rapidly mix transversely and then disperse longitudinally, presumably driven by the Taylor dispersion mechanism, [58,59]. Although Zhang and Frigaard [69] have considered dispersion of such fluids in laminar regimes, for laminar flows primary cementing does not typically fall into the Taylor-regime.

Axial dispersion in turbulent flows of Newtonian fluids was initially studied by Taylor [59]. Upon applying the Reynolds analogy to model the turbulent dispersivity, he then integrated the relative velocity profile across the pipe to calculate the axial bulk dispersivity. Taylor used tabulated data from the *universal* distribution of velocity which is known to be valid only at high Reynolds number and therefore his results significantly deviate from experimental data [12,33,61]. Taylor's analysis was later revisited by Tichacek et al. [61] and Flint and Eisenklam [16] who utilized experimental velocity profiles to give better estimates. Nonetheless, both these studies deviate from experimental results at low Reynolds number ($Re < 10^4$) mainly because the experimental velocity profile was unable to capture the wall layer. In another study Ekambara and Joshi [12] estimated the axial dispersion with a velocity profile obtained computationally using the $k-\epsilon$ model. A comparison of these approaches with the experimental data can be found in Hart et al. [33].

Alternative approaches to that of Taylor can be found in the literature. For example, Aris [3] developed a concentration moment equation which described the distribution of solute. Chikwendu [6] divided the flow into N well mixed zones of parallel flows and found the dispersion of each zone separately, then solving the N coupled dispersion equations to give an estimate of the dispersion coefficient. Hart et al. [33] compared the results of this method

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with their experimental data and the results of Taylor. Dispersion in unsteady problems has been studied by Gill and Sankarasubramanian [21], Sankarasubramanian and Gill [53], Vedel and Bruus [63] and others. Other Taylor dispersion studies have focused on natural flows, e.g. Fischer [15], Day [7].

For inelastic non-Newtonian fluids, axial dispersion in laminar [1,2,5,69] and turbulent [39,57,64] flows has been studied. In the case of turbulent regimes, Krantz and Wasan [39], Wasan and Dayan [64] studied dispersion of power-law fluids using the turbulent velocity profile of Bogue and Metzner [4]. Wasan and Dayan [64] predicted the axial dispersion to increase with Reynolds number, contradicting Taylor's model for dispersion. Krantz and Wasan [39] modified the earlier results by adding a wall layer to the velocity profile. However, the validity of their results is questionable since the velocity scale used appears to be different from that of Bogue and Metzner [4].

As noted by Ekambara and Joshi [12], Hart et al. [33], Krantz and Wasan [39], Tichacek et al. [61], good estimation of the Taylor dispersion demands an accurate velocity profile. Laminar velocity profile are integrable from the constitutive law, and the Metzner-Reed generalised Reynolds number provides an economical description of the hydraulic closure relationship. Hydraulic-style calculations for turbulent shear-thinning and yield stress fluids have studied since the 1950's; see e.g. [26–28,31,32,52]. Although not universally accepted, the phenomenological method of Dodge-Metzner-Reed [9,40] is popular in many process industries. In this method a generalised Reynolds number is defined based on the local power-law parameters. Then, a closure relationship is established for the frictional pressure drop as a function of the generalised Reynolds number, calibrated with the available data. The Dodge-Metzner-Reed approach was intended to apply to all generalised Newtonian fluids. The extension to yield stress fluids can be found in [17,46,48], as well as internally within technical literature of many petroleum companies. Tests against experimental data are described by [23]. More recently, comparisons with direct numerical simulation data were made by [51].

In the context of dispersion the Dodge-Metzner-Reed approach is attractive in that the hydraulic calculations (and closure) are linked to a universal log-law velocity profile, proposed by Dodge and Metzner [9]. Such profiles may be used directly to calculate Taylor dispersion coefficients. However, two common deficiencies occur: (i) the log-law is not valid at the centreline of the pipe/duct; (ii) the log-law must be matched/patched to a different velocity approximation close to the wall. Various centreline corrections have been suggested, including the correction of [49] and exponential correction of [4]. Near the wall, Krantz and Wasan [38] argued that Reynolds stresses decay as the cube of the distance, and therefore suggested that the wall layer effect could be significant. Krantz and Wasan [39] developed the analysis framework to evaluate the wall layer for power-law fluids.

In this paper we consider dispersion of yield stress fluids. In laminar flows, increasing the yield stress tends to flatten the velocity profile and hence reduce Taylor dispersion. In turbulent flows it is generally thought that the yield stress has little influence on the velocity profile in the turbulent core, but is known to retard turbulent transition. Equally, since the yield stress contributes to the effective viscosity we might expect that wall-layer effects are significant as the yield stress increases. Hence the interest in weak turbulence where wall-layers are thicker and occupy a larger proportion of the duct area, also where the velocity changes are greatest. Our study explores the subtlety of this relationship.

An outline of our paper is as follows. In Section 2 we outline the dimensionless numbers and hydraulic calculation for pipe flows of Herschel-Bulkley fluids. This leads in Section 3 to the turbulent velocity profile, corrected at the centreline and wall. Using Reynolds analogy we find the turbulent diffusivity and finally we give estimates for the Taylor dispersion coefficient. In Section 4 we outline analogous results and analysis for channel flows (modelling a section of the narrow annulus in cementing). The paper is closed with a discussion and conclusions in Section 5.

2. Pipe flow

Consider fully developed steady flow of a Herschel-Bulkley fluid along a pipe. The axial momentum balance relates the axial gradient of frictional pressure $\hat{p}_f$ to the wall shear stress $\hat{\tau}_w$, which is then described in terms of the inertial stress scale $\hat{\rho}\hat{W}_0^2/2$ and (Fanning) friction factor f_f:

$$-\frac{\hat{D}}{4}\frac{\partial \hat{p}_f}{\partial \hat{z}} = \hat{\tau}_w = \frac{\hat{\rho}\hat{W}_0^2}{2} f_f, \tag{1}$$

where $\hat{W}_0$ is the mean velocity and $\hat{\rho}$ is the fluid density.[2] Herschel-Bulkley fluids are defined rheologically by three parameters: the yield stress $\hat{\tau}_Y$, the consistency $\hat{\kappa}$, and the power law index n. In the hydraulic calculations that are generally performed, the fluid properties: $\hat{\rho}$, $\hat{\tau}_Y$, $\hat{\kappa}$, n, and the pipe diameter $\hat{D}$ are known. The aim is to define the closure relationship between the wall shear-stress $\hat{\tau}_w$ and the mean velocity $\hat{W}_0$ for the different flow regimes.

A widely used approach is that of Dodge and Metzner [9] in defining f_f as a function of the generalised (Metzner-Reed) Reynolds number and power law index, with an additional dimensionless parameter needed to quantify yield stress effects. Although we are concerned with turbulent flows, the Metzner-Reed approach requires the laminar flow relations. The Metzner-Reed generalized Reynolds number is defined:

$$Re_{MR} = \frac{8\hat{\rho}\hat{W}_0^2}{\hat{\kappa}'(\hat{\dot{\gamma}}_N)^{n'}} \tag{2}$$

where the primed variables are:

$$\hat{\kappa}' = \frac{\hat{\tau}_w}{(\hat{\dot{\gamma}}_L)^{n'}}, \qquad n' = \frac{\mathrm{d}\ln\hat{\tau}_w}{\mathrm{d}\ln\hat{\dot{\gamma}}_L}. \tag{3}$$

The Newtonian strain rate at the wall is $\hat{\dot{\gamma}}_N$ and $\hat{\dot{\gamma}}_L$ is the laminar strain rate:

$$\hat{\dot{\gamma}}_N = \frac{8\hat{W}_0}{\hat{D}}, \qquad \hat{\dot{\gamma}}_L = \frac{8\hat{W}_L}{\hat{D}}. \tag{4}$$

The velocity $\hat{W}_L$, used to define $\hat{\dot{\gamma}}_L$, is the mean velocity that the fluid would have in a laminar flow, driven by the wall shear-stress $\hat{\tau}_w$. Note that $\hat{W}_L$ and $\hat{\dot{\gamma}}_L$ are defined by the wall shear stress $\hat{\tau}_w$ across all flow regimes, but will only equal $\hat{W}_0$ and $\hat{\dot{\gamma}}_N$ in the case that the flow is laminar.

For laminar flows, the Buckingham-Reiner equation can be derived, which is an algebraic equation relating the flow rate to the wall shear stress. The Rabinowitsch-Mooney procedure results in the same expression. For Herschel-Bulkley fluids the result is:

$$\hat{\dot{\gamma}}_L = \frac{4n}{3n+1}(1-r_Y)^{1/n+1}\left[\frac{\hat{\tau}_w}{\hat{\kappa}}\right]^{1/n} \times \left[(1-r_Y)^2 + \frac{2(3n+1)(1-r_Y)r_Y}{2n+1} + \frac{(3n+1)r_Y^2}{n+1}\right]. \tag{5}$$

Here $r_Y = \hat{\tau}_Y/\hat{\tau}_w$, which also represents the dimensionless radial position of the yield surface. Combining (3) with (5) we find:

$$n' = n(1-r_Y)\frac{(n+1)(2n+1)+2n(n+1)r_Y+2n^2r_Y^2}{(n+1)(2n+1)+3n(n+1)r_Y+6n^2r_Y^2+6n^3r_Y^3}, \tag{6}$$

[2] In this paper we denote dimensional quantities with a ˆ symbol and dimensionless quantities without.