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A decoupling multiple-relaxation-time lattice Boltzmann flux solver for non-Newtonian power-law fluid flows

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ABSTRACT

This work presents a decoupling multiple-relaxation-time lattice Boltzmann flux solver (MRT-LBFS) for simulating non-Newtonian power-law fluid flows. The decoupling MRT-LBFS is a finite volume solver for the direct update of fluid variables at cell centers. Its fluxes at each cell interface are modeled physically in a mesoscopic way through local reconstruction of the MRT-LBM solutions of density distribution functions (DDFs). In particular, inviscid and viscous fluxes are simultaneously obtained through lattice summations of equilibrium and non-equilibrium DDFs. Following the MRT model, non-equilibrium DDFs are evaluated in the moment space by using the relationships given from the Chapman–Enskog analysis so that collisional invariant properties of conserved variables can be effectively incorporated into the flux reconstruction process. Unlike most existing LB models, in which the relaxation time depends on fluid viscosity, the present method completely decouples the mutual dependence of the relaxation time and viscosity so that the relaxation time can be selected freely. Several numerical examples of non-Newtonian power-law fluid flows, including plane Poiseuille flow in a channel, lid-driven cavity flows and polar-cavity flows in a sector, have been simulated for validation. The obtained results compare well with the benchmark data. It has been shown that the decoupling MRT-LBFS has second order of accuracy in space and can be effectively applied to simulate non-Newtonian flows on non-uniform grids.

1. Introduction

Non-Newtonian fluid flows are frequently encountered in many physical and industrial processes [1], such as porous flows of oils and gases [2], biological fluid flows of blood [3], saliva and mucus, penetration grouting of cement mortar and mixing of massive particles and fluids in drug production [4]. As compared with Newtonian fluid flows, the constitutive behavior of non-Newtonian fluid flows is usually more complex and highly non-linear, which may bring more difficulties in using numerical methods [5] to study such flows. For instance, due to the dependence of viscosity on shear rate, accurate and efficient numerical methods are required for evaluation of velocity gradients, which may be very large and have sharp changes in some circumstances. Nevertheless, to effectively study non-Newtonian fluid flows, a number of macroscopic and mesoscopic numerical methods have been proposed [6–9]. Among them, the mesoscopic lattice Boltzmann method [9–14], which has been proven to be a simple yet efficient solver for simulating a variety of complex fluid flows, is receiving more attention due to its distinguishing features. Several variants of LBM [9,13–21] have been proposed and improved to effectively consider the complexity of constitutive equations for non-Newtonian fluids.

Based on the single-relaxation-time (SRT) model, the earliest LBM for simulation of non-Newtonian fluid flows was perhaps proposed by Aharonov and Rothmans [9]. Although the SRT model is simple for implementation, its numerical stability is largely affected by the value of the relaxation parameter τ [11], which is coupled together with the viscosity μ of the considered fluid, that is, $\mu = \rho c_s^2(\tau - 1/2)$. In particular, as τ approaches 1/2 or becomes too large, numerical instability of the SRT model may occur. For non-Newtonian fluid flows, small viscosity can be common since it is a local parameter and passively determined by the shear rate in the flow field. As a consequence, numerical instability of the LBM can be very critical in simulating non-Newtonian fluid flows. To alleviate this defect, Gabbanelli et al. [15] proposed an improved LBM based on the work of Aharonov and Rothmans [9] by artificially imposing lower and upper bounds to the fluid viscosity.

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Their improved model has been successfully validated by simulating channel flows and reentrant flows. However, the appropriate lower and upper bounds may be difficult to determine and limit its applications to flows in a narrow range of shear-dependent viscosities. To remove this drawback, Boyd et al. [16] proposed a second-order lattice Boltzmann model by applying the non-equilibrium terms of DDFs to compute the shear rate. Yoshino et al. [17] also devised an improved model by introducing the shear stress into the equilibrium DDF so that the relaxation time in their model can be decoupled from fluid viscosity. Vikhansky [18] proposed a new model for yield-stress fluids, in which collisions are treated implicitly. Later, Wang et al. [19] argued that the LB model proposed by Yoshino et al. [17] does not satisfy mass conservation law and further proposed a mass-conserving model by incorporating shear stress into the equilibrium DDF. In their model, the shear rate is evaluated through non-equilibrium DDFs. However, due to the inclusion of shear stress into the equilibrium DDF, the computation of the shear rate and the equilibrium DDF at each time step is implicitly coupled. As compared with SRT-LBM [15–19], only a few MRT-LBMs [20,21] were proposed for simulating generalized non-Newtonian fluid flows. Numerical results showed that the MRT-LBM is able to maintain better numerical stability.

It is also noticed that the coupling issue between the relaxation parameter and fluid viscosity has not been fully resolved, and effective simulation of non-Newtonian fluid flows with curved boundaries by the LBM may be still challenging. In particular, to the best of our knowledge, there is few MRT lattice Boltzmann models that decouple the dependence of viscosity and relaxation time. In addition, the existing SRT and MRT lattice Boltzmann models are proposed based on the standard LBM and applied to flows with straight walls. Although they retain the distinguishing merits of LBM, the drawbacks of the standard LBM are also kept, such as limitation on uniform grids, tie-up between time step and grid spacing and requirement of substantial virtual memory [22]. These drawbacks can be effectively removed by the recently proposed lattice Boltzmann flux solver (LBFS) [23–25], which is a finite volume solver based on the standard LBM for direct update of conservative variables at cell centers. LBFS reconstructs its fluxes at cell interfaces locally in a mesoscopic way through lattice Boltzmann solutions. Existing LBFSs [22–25] are based on the SRT lattice Boltzmann model for simulating different Newtonian fluid flows, such as isothermal flows, thermal flows and multi-phase flows.

In this work, a decoupling LBFS is proposed based on the MRT lattice Boltzmann model for effective simulation of non-Newtonian power-law fluid flows. Unlike existing decoupling SRT models, which introduce the constitutive equation for shear stress into the equilibrium DDFs, the present method is based on the original MRT-LBM but effectively decouples the relaxation time and viscosity. To achieve this goal, a Chapman–Enskog analysis of the MRT-LBM is carried out to derive the relationships between the macroscopic fluxes and mesoscopic DDFs. When the derived relationships are applied directly to reconstruct the macroscopic fluxes, the coupling issue remains unresolved. Alternatively, according to the gas kinetic theory, the shear rate is directly related to non-equilibrium terms of the non-conserved variables in moment spaces for the MRT-LB model, whose relationships have been verified and applied in many LBM applications [20]. It may be noted that these relationships can also be applied directly to approximate the fluid viscosity and viscous fluxes in the LBFS. The evaluation of these non-equilibrium terms is carried out in the moment space by applying the formula given from the Chapman–Enskog analysis and collisional invariant features. The proposed method will be validated through several flow problems of non-Newtonian power-law fluids, including plane Poiseuille flow in a channel, lid-driven cavity flows and polar cavity flows.

2. Mathematical model for non-Newtonian power-law fluid flows

In this work, our attention will be focused on two-dimensional incompressible non-Newtonian power-law fluid flow. The mathematical model of this flow can be written as [20]:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{u}) = 0 \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla\cdot(\rho \boldsymbol{u}\boldsymbol{u}) + \nabla p - \nabla\cdot\boldsymbol{\tau} = \mathbf{0} \tag{2}$$

where ρ, $\boldsymbol{u}$ and p are respectively the fluid density, velocity and the pressure; $\boldsymbol{\tau}$ is the shear stress determined by the constitutive equation of the power-law fluid:

$$\boldsymbol{\tau} = \mu\left(|\dot{\boldsymbol{\gamma}}|\right)\dot{\boldsymbol{\gamma}} \tag{3}$$

Here, $\dot{\boldsymbol{\gamma}}$ is the shear rate defined by

$$\dot{\boldsymbol{\gamma}} = \nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T, \tag{4}$$

and $|\dot{\boldsymbol{\gamma}}|$ is defined by $|\dot{\boldsymbol{\gamma}}| = \sqrt{(\dot{\boldsymbol{\gamma}} : \dot{\boldsymbol{\gamma}})/2}$. For the non-Newtonian power-law fluid considered in this work, the effective viscosity $\mu(|\dot{\boldsymbol{\gamma}}|)$ is a non-linear function of the shear rate $\dot{\boldsymbol{\gamma}}$:

$$\mu\left(|\dot{\boldsymbol{\gamma}}|\right) = \mu_{PL}|\dot{\boldsymbol{\gamma}}|^{n-1} \tag{5}$$

In Eq. (5), μ_{PL} and n are two modeling parameters, respectively known as the flow consistency coefficient and power-law index. When $n<1$, Eq. (5) models a shear thinning or pseudo-plastic fluid. When $n>1$, it models shear thickening or dilatant fluid, and when $n=1$, it represents a Newtonian fluid.

3. MRT-LB model and Chapman–Enskog expansion analysis

3.1. MRT-LB model

The lattice Boltzmann equation with multi-relaxation-time (MRT) based-BGK approximation can be written as [11,20]:

$$\mathbf{f}(\mathbf{x} + \mathbf{e}_\alpha \delta_t, t + \delta_t) - \mathbf{f}(\mathbf{x}, t) = -\boldsymbol{M}^{-1}\boldsymbol{S}\boldsymbol{M}\left[\mathbf{f}(\mathbf{x}, t) - \mathbf{f}^{(eq)}(\mathbf{x}, t)\right], \quad \alpha = 0, 1, \ldots, N, \tag{6}$$

where $\mathbf{x}$ represents a physical location; δ_t is the streaming time step and $\mathbf{e}_\alpha$ is the particle velocity in the α direction; N is the number of discrete particle velocities; $\mathbf{f}$ and $\mathbf{f}^{(eq)}$ are the discrete DDF and its corresponding equilibrium state, which can be written as:

$$f_\alpha^{(eq)}(\mathbf{x}, t) = \rho w_\alpha\left[1 + \frac{\mathbf{e}_\alpha\cdot\mathbf{u}}{c_s^2} + \frac{(\mathbf{e}_\alpha\cdot\mathbf{u})^2 - (c_s|\mathbf{u}|)^2}{2c_s^4}\right] \tag{7}$$

Here, the coefficients w_α and the sound speed c_s are respectively given as $w_0 = 4/9$, $w_{1\sim4} = 1/9$, $w_{5\sim8} = 1/36$ and $c_s = c/\sqrt{3}$. In 2D case, the D2Q9 lattice velocity model is used and can be written as

$$\mathbf{e}_\alpha = \begin{cases} 0 & \alpha = 0 \\ (\cos[(\alpha-1)\pi/2], \\ \quad \sin[(\alpha-1)\pi/2])c & \alpha = 1,2,3,4 \\ \sqrt{2}(\cos[(\alpha-5)\pi/2+\pi/4], \\ \quad \sin[(\alpha-5)\pi/2+\pi/4])c & \alpha = 5,6,7,8 \end{cases} \tag{8}$$

Here $c = \delta_x/\delta_t$, δ_x is the lattice spacing. The macroscopic density ρ and momentum $\rho\mathbf{u}$ are computed by the first and second-order lattice moments of $\mathbf{f}$

$$\rho = \sum_{\alpha=0}^{N} \mathbf{f}_\alpha \text{ and } \rho\mathbf{u} = \sum_{\alpha=0}^{N} \mathbf{f}_\alpha \mathbf{e}_\alpha \tag{9}$$