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Flow and displacement of waxy crude oils in a homogenous porous medium: A numerical study

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ABSTRACT

In the present study, the displacement of waxy crude oils is numerically/theoretically investigated in the water-flooding operation. The oil was assumed to obey the Houska model–a robust thixotropic fluid model which is often realized to well describe the rheology of waxy oils in different parts of the world. Based on the concept of *effective viscosity,* a modified version of the Darcy's law was developed for this particular fluid model in order to describe its flow through a homogenous porous medium. Use was made of numerical and theoretical methods to study the displacement of Houska fluid by water in two benchmark problems: (i) the Buckley–Leverett problem, and (ii) the five-spot problem. It was found that the yield stress of the Houska fluid being *variable* (i.e., shear- and time-dependent) has a retarding effect on the water breakthrough phenomenon. The breakdown-to-rebuild ratio in the Houska model was shown to play a key role in the water breakthrough phenomenon provided that it is very large. At this extreme, however, the effect was attributed mostly to the shear-thinning behavior of the Houska fluid rather than its thixotropic behavior. In fact, at sufficiently low breakdown-to-rebuild ratios (i.e., when fluid's thixotropy becomes progressively more important) it had no significant effect on the water breakthrough phenomenon. Therefore, it is concluded that in competition with shear-thinning, the thixotropic behavior of Houska fluid plays a secondary role, if any, in the water-flooding operation.

1. Introduction

Liquid/Liquid displacement is encountered in many branches of engineering. One can mention, for example the water flooding operation widely used in the oil industry [1]. The operation typically involves injection of water into a partially-depleted reservoir in order to direct the oil left in the rock pores towards the production well. In practice, however, due to a difference between their mobility, sooner or later, the injected water reaches the production well. An estimation of the time needed by the water to breakthrough is of paramount importance in the oil industry as it directly affects the oil throughput. The same is true for the sweep efficiency (i.e., the volume of the oil recovered by the water) which is important as well. Due to the difficulty of carrying out experimental studies under real reservoir conditions, theoretical and/or numerical methods are often used for their estimation. But, this is not an easy task realizing the fact that we are dealing with the flow of a complex fluid through a porous medium (i.e., the oil reservoir). A review of the literature on simulating fluid flow through porous media reveals that there are different approaches for modeling such flows [2]. One can particularly mention the promising and more accurate pore-scale and network models [2]. But, due to hardware limitations and also the uncertainties normally involved at pore-scale level, such models are still far from becoming fully-operational. In fact, the *continuum approach* appears to be still favored in industry and academia alike for this purpose–thanks to its simplicity and adequate accuracy [3–5]. In this approach, a porous medium is described by macroscopic properties such as porosity and permeability. The flow rate is then related to the pressure gradient through the Darcy's law. This methodology works fine for ordinary crude oils, which are known to be Newtonian. Waxy crude oils, on the other hand, are known to behave as non-Newtonian fluids at sufficiently low temperatures. The Darcy's law must be modified for such complex oils before it can be invoked in any meaningful theoretical and/or numerical study. The idea is to replace the Newtonian viscosity in this law by an appropriate non-Newtonian *effective viscosity* for the fluid under investigation.

The idea of *effective viscosity* has been shown to work well for generalized Newtonian fluids such as the Herschel–Bulkley model [6–10]. The problem is that the yield stress and viscosity of waxy

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crude oils is *variable* (i.e., shear- and time-dependent). That is to say that, waxy crude oils are thixotropic in addition to being viscoplastic [11,12]. Pearson and Tardy [13], and Pritchard and Pearson [14] appear to be the only researchers who have tried to extend the Darcy's law to thixotropic fluids. However, the thixotropic model adopted by them (namely, the modified Bautista–Manero model [15]) is not a good model for representing viscoplastic waxy crude oils. In fact, as far as we are aware, Darcy's law has not previously been extended to any rheological model representing any real waxy crude oil. This is certainly true for the Houska model [16], which is known to well describe the rheology of waxy crude oils in many parts of the world. With this in mind, in the present work we intend to first extend the classic Darcy's law to this particular fluid model and then use it for investigating its water-flooding performance in two benchmark problems: (i) the one-dimensional Buckley–Leverett problem [17], and (ii) the two-dimensional five-spot problem [18]. The main objective of the work is to see how the material parameters in the Houska model affect its breakthrough time and sweep efficiency in the water-flooding operation.

To reach its objectives, the work is organized as follows: In the next section, we formulate the mathematical framework for our fluid-mechanics problem by developing a modified Darcy's law for the Houska fluid. We also describe the benchmark problems mentioned above in more details together with introducing the initial and boundary conditions required to close the problem. We then proceed with describing the numerical method of solution in some details. Numerical results are presented next together with offering our understanding of their significance. The work is concluded by highlighting its major findings.

2. Mathematical formulation

We consider the pressure-driven flow of a waxy oil in a porous medium of uniform permeability and porosity. Although three-dimensional models are often used in the oil sector for simulating the characteristics of real reservoirs [19,20], simplified one- and/or two-dimensional models are still in widespread use for elucidating the mechanisms involved in the recovery process. One such model, which was introduced by Buckley and Leverett in 1941 [17], tries to describe the displacement of immiscible fluids (for example, water/oil) in sand reservoirs. In their seminal paper, Buckley and Leverett [17] introduced a simplified mass-balance-equation (MBE) to model the frontal advance in the one-dimensional flow through a constant-area reservoir. They used fractional flow as a function of water saturation for this purpose having assumed that the oil and water are both Newtonian fluids. To simplify the analysis, they neglected the capillary pressure. As to the initial condition, it is assumed that the reservoir is initially filled with the oil. As the boundary condition(s), it was assumed that water is being injected (with a known rate) at one end with the oil being produced (at a constant rate) at the other end. Having analytically solved the governing equations, Buckley and Leverett [17] determined the breakthrough time and sweep efficiency through determining the speed of the waterfront. Fig. 1 shows a typical waterfront which is propagating from left to right in such a simplistic oil reservoir.

Another benchmark problem which is in common use in the oil industry is the homogenous quarter five-spot problem [18]. It is a two-dimensional water-flooding test case in which the production well is surrounded by four injection wells in a repeating pattern. Fig. 2 shows the location of the injection/production wells in such an arrangement. In practice, water is injected from the injection wells and pushes the oil towards the production wells. For this benchmark problem, we rely on the water saturation profiles in order to determine the water breakthrough time.

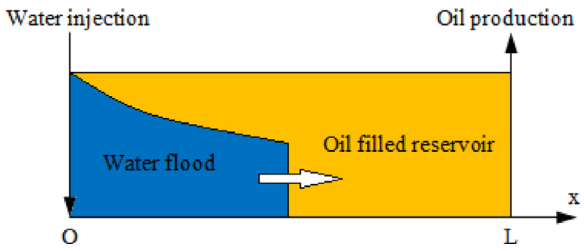

Fig. 1. Frontal advance of the injected water in the one-dimensional Buckley–Leverett problem.

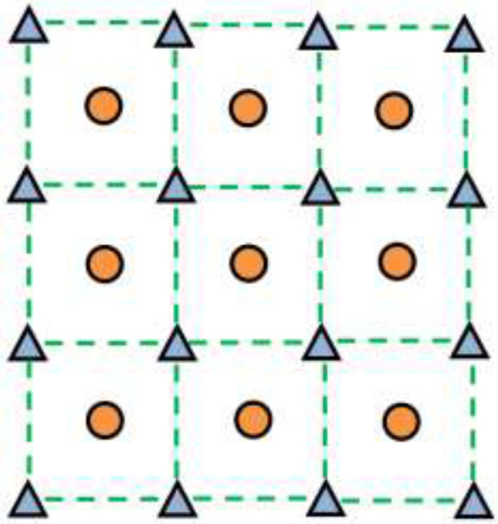

Fig. 2. The five-spot problem used for simulating the water flooding operation (circles refer to the production wells with triangles referring to the injection wells).

The starting point for a mathematical treatment of the liquid/liquid displacement in the two benchmark problems mentioned above is the conservation of mass for each phase. For two immiscible liquids flowing simultaneously through a porous medium, the mass balance for each phase can be written as [20]:

$$\frac{\partial}{\partial t}(\phi \rho_\alpha S_\alpha) + \nabla.(\rho_\alpha \mathbf{u}_\alpha) = q_\alpha\,;\ \alpha = w, o \tag{1}$$

where t is the time, ϕ is the reservoir's porosity, ρ is the density of each phase, S is the phase saturation, $\mathbf{u} = (u, v)$ is the phase velocity vector, and q is the rate of injection/production of each phase. The subscript α in the above equation refers to each phase with "w" denoting the displacing liquid (say, water) and "o" denoting the displaced liquid (say, oil). For Newtonian/Newtonian displacement, phase velocities are often calculated using the classic Darcy's law. For non-Newtonian fluids, in general, this law is not valid in its original form. For generalized Newtonian fluids (GNF), however, this law can be extended by simply replacing the *Newtonian* viscosity by an *effective viscosity*. With this in mind, for the displaced fluid we can write [20]:

$$\mathbf{u}_o = -k\frac{k_{ro}}{\mu_{eff,o}}\nabla \mathrm{p}, \tag{2}$$

where p is the isotropic pressure, k is the reservoir's permeability, k_{ro} is the relative permeability of the oil phase (which is assumed to be a function of the phase saturation, S), and $\mu_{eff,\,o}$ is the *effective viscosity* of the waxy oil. For the displacing Newtonian fluid,