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Short Communication

Mobility and pore-scale fluid dynamics of rate-dependent yield-stress fluids flowing through fibrous porous media

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ABSTRACT

The steady flow of viscoplastic fluids through fibrous porous media is studied numerically and theoretically. We consider fluids with a plastic yield stress and a rate-dependent viscosity that can be described by the Herschel-Bulkley model. We first investigate the pore-scale flow characteristics through numerical simulations for flow transverse to a square array of fibers with comprehensive parametric studies to independently analyze the effects of the rheological properties of the fluid and the geometrical characteristics of the fibrous medium. Our numerical simulations show that the critical Bingham number at which the flow transitions from a fully-yielded regime to locally unyielded regions depends on the medium porosity. We develop a scaling model for describing the bulk characteristics of the flow, taking into account the coupled effects of the medium porosity and the fluid rheology. This model enables us to accurately predict the pressure-drop–velocity relationship over a wide range of Bingham numbers, power-law indices, and porosities with a formulation that can be applied to a square or a hexagonal array of fibers. The ultimate result of our scaling analysis is a generalized form of Darcy's law for Herschel-Bulkley fluids with the mobility coefficient provided as a function of the system parameters. Based on this model, we construct a modified Bingham number rescaled with a suitable porosity function, which incorporates all the rheological and pore-scale parameters that are required to determine the dominant flow regime.

1. Introduction

Flows of complex fluids through porous media are encountered in many industrial processes such as polymer processing, filtration and enhanced oil recovery operations [1]. Understanding the flow characteristics in these processes is challenging due to the complexity of the fluid behavior in the porous micro-structure. One of the most important objectives in studying porous media flows is to find the relationship between the macroscopic imposed pressure drop and the flow rate [2]. For Newtonian fluids, this relationship is described by Darcy's law, which shows a linear proportionality between the flow rate and the pressure drop across a porous medium. Complex fluids do not typically obey Darcy's law due to nonlinearities in their rheological response to an applied stress. However, it is possible to develop modified forms of Darcy's law for generalized Newtonian fluids [3]. This involves evaluating an effective or characteristic fluid viscosity η_{eff}, which depends on the constants in the relevant constitutive model as well as the flow rate and the microstructural characteristics of the porous medium. Consequently, the modified Darcy law for the flow of generalized Newtonian fluids through porous media is of the form

$$\frac{\Delta p}{L} = \frac{\eta_{\mathrm{eff}} U}{\kappa} \tag{1}$$

where $\Delta p/L$ is the pressure drop per unit length of the porous medium, U is the superficial or apparent velocity of the fluid, and κ is the permeability of the porous medium.

There are several studies on the flow of generalized Newtonian fluids through porous media, many of which consider the flow of shear-thinning fluids (without a yield stress). Examples of these studies include experimental investigations of the flow through packed spheres [4–6] and Hele-Shaw cells (designed to capture relevant characteristics of a porous medium) [7,8], as well as numerical studies [9–12] and pore-network modeling [13,14].

However, many complex fluids such as filled polymer melts, foams, and emulsions also exhibit a yield stress; therefore, a critical imposed pressure drop typically needs to be reached before the material can flow through the porous medium. Among the earliest studies that considered a critical pressure gradient flow through porous media are the work by Gheorghitza [15] and by Entov [16], in which theoretical study of Bingham fluids was presented. Chevalier and Talon [17] recently used a lattice-Boltzmann scheme to numerically study the two-dimensional flow of Bingham fluids through porous media.

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Inelastic materials with a yield stress and a rate-dependent viscosity are often described by the Herschel-Bulkley (HB) model [18], which captures both salient features and we use this model in the present study. Park [19] experimentally investigated steady flow of HB fluids through packed beds of spheres and reported large deviations from semi-empirical models developed for Newtonian fluids. Al-Fariss and Pinder [20] were the first to present an empirical model for describing the macroscopic flow behavior of yield stress fluids in porous media in the form of a modified Darcy's law. Using the HB model, they modified the Blake-Carman-Kozeny equation to describe the flow of waxy oils through beds of packed spheres. Liu and Masliyah [21] studied HB fluids in ducts of arbitrary cross-sections and correlated the pressure drop to the flow rate using three fitting constants for each duct shape. Then using a volume averaging technique, they developed a suitable bundle-of-capillaries model. More recent studies on the flow of HB fluids include the work by de Castro et al. [22], who also used a bundle-of-capillaries model and matched the numerical simulations with experimental results for the flow of a xanthan gum solution in order to deduce the pore size distribution in a porous medium. It has been argued that capillary (bundle-of-tubes) models cannot describe the flow of HB fluids through porous media for a wide range of shear-thinning indices [23]. Balhoff [24] and Sochi [25] have used pore-scale network modeling to study the flow of yield stress fluids through porous media. A recent experimental study of HB fluids in porous media was presented by Chevalier et al. [26]. They measured the pressure drop vs. flow rate for the flow of a yield stress fluid (water-in-oil emulsion) through packed glass beads and proposed an empirical relationship for a medium porosity of 33%.

Most of the literature on the flows of complex fluids through porous media discuss the flow through packed beds of spherical particles. There are fewer studies that consider complex fluids flow through fibrous porous media. The anisotropic characteristics of fibrous media leads to a different functional form for the pressure drop dependence on the medium porosity compared to that of packed spheres. Moreover, studies on packed spheres typically only cover a narrow range of porosities close to those expected for random close packing; thus they cannot be applied for arbitrary fibrous media, where a much wider porosity range is accessible. Bleyer and Coussot [27] have investigated the flow of HB fluids in fibrous media using numerical simulations of flow through an ordered array of cylinders. Our numerical approach in this work is similar to that of Bleyer and Coussot. However, the focus of their study was on the velocity fields and demonstration of low sensitivity of velocity fields to the fluid rheology; while it is known that the rheological properties significantly affect the fluid mobility, which is the focus of our work here.

In this Short Communication, we investigate the flow of purely viscous rate-dependent yield stress fluids through fibrous media by means of numerical simulations and a scaling analysis. By comparing the two approaches, we develop an effective viscosity function from our scaling model, which can be used in the generalized Darcy's law for steady fully-developed flow of HB fluids through fibrous media.

2. Rate-dependent mobility

In flows of complex fluids through porous media, the fluid effective viscosity, η_{eff}, is a function of the medium porosity because the pore-scale shear rate in the fluid between the adjacent fibers or spheres also varies with the volume fraction of solid packing. Hence, the effective viscosity can be combined with the medium permeability, κ, (which is also a function of the porosity) to yield a single coefficient that relates the pressure drop across the bed to the superficial velocity in the modified Darcy's law (Eq. (1)). This single coefficient is called the *fluid mobility* (with units of $[\text{m}^2\text{Pa}^{-1}\text{s}^{-1}]$) and is defined as [28]:

$$M \equiv \frac{\kappa(\varepsilon, d)}{\eta_{\text{eff}}(\varepsilon, \dot{\gamma}_{\text{eff}}, \text{fluid rheology})} \tag{2}$$

where ε is the porosity and $\dot{\gamma}_{\text{eff}}$ is the effective shear rate, which is a function of the flow rate and the microstructure of the medium. We consider a regularized bi-viscosity formulation of the Herschel-Bulkley (HB) model, in which the viscosity can be written in the following form

$$\eta(\dot{\gamma}) = \begin{cases} \eta_r & \sigma < \sigma_y \\ \dfrac{\sigma_y}{\dot{\gamma}} + m\dot{\gamma}^{n-1} & \sigma \geq \sigma_y \end{cases} \tag{3}$$

where σ_y, n, and m denote the fluid yield stress, the power-law exponent, and the consistency of the fluid, respectively. The characteristic shear rate $\dot{\gamma}$ is defined as $\dot{\gamma} = \sqrt{\frac{1}{2}\left(\dot{\boldsymbol{\gamma}} : \dot{\boldsymbol{\gamma}}\right)}$ where $\dot{\boldsymbol{\gamma}} = \nabla\mathbf{u} + \nabla\mathbf{u}^T$. The theoretical viscosity for a true HB fluid at stresses below the yield/critical stress is $\eta_r \to \infty$. In the regularized model, below the yield stress, the viscosity is taken to be constant at a very large value denoted by η_r, and the crossover takes place at a critical shear rate $\dot{\gamma}_c$ given by $\sigma_y/\dot{\gamma}_c + m\dot{\gamma}_c^{n-1} = \eta_r$.

Four dimensionless groups that determine the fluid mobility are the Reynolds number, the Bingham number (i.e. the ratio of the fluid yield stress to a characteristic viscous stress [29]), the medium porosity ε, and the power-law exponent n. The Bingham number for a rate-dependent HB fluid can be defined as

$$\text{Bn} \equiv \frac{\sigma_y}{m(U/d)^n} \tag{4}$$

where d is the characteristic length scale of the flow. For a fibrous medium, the natural choice for this length scale is the average fiber diameter. We also use a generalized Reynolds number defined as $\text{Re} = \rho U^{2-n} d^n / m$ to take into account the rate-dependent inertial effects in the fluid. In the present communication, we study the low Reynolds number regime (where the fluid inertia is negligible), i.e. $\text{Re} \ll 1$ and we investigate the coupled and interdependent effects of the three other dimensionless numbers (Bn, ε, and n) on the flow of HB fluids transverse to a periodic array of fibers.

3. Numerical analysis

The conservation of mass and momentum for the steady-state flow of generalized Newtonian fluids are given by Eqs. (5) and (6) respectively.

$$\nabla . \mathbf{u} = 0 \tag{5}$$

$$\rho\mathbf{u} . \nabla\mathbf{u} = -\nabla p + \nabla . \left(\eta(\dot{\gamma})\left\{\nabla\mathbf{u} + \nabla\mathbf{u}^T\right\}\right) \tag{6}$$

We numerically solve these equations for transverse flow through aligned fibrous media, building on the approach that we adopted for steady slow of Carreau fluids [30]. The idealized domain that we consider here for representing a fibrous medium consists of a periodic array of cylinders in a square arrangement as shown in Fig. (1). It is also straightforward to extend this analysis to hexagonal arrangements [30]. We assume steady two-dimensional flow and take advantage of the symmetry and periodicity in the system to solve the equations in a representative unit cell as shown in Fig. 1 (bottom). COMSOL Multiphysics 4.3a is used to build the numerical model and solve the equations using a finite-element technique. Depending on the relative spacing s/d, varying numbers of mesh elements ($10^4 - 10^5$) were used to achieve mesh-independent results. In the unit cell shown in Fig. 1, the imposed boundary conditions consist of zero slip velocity on the cylinder