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The importance of rheology characterization in predicting turbulent pipe flow of generalized Newtonian fluids

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ABSTRACT

Most Direct Numerical Simulation (DNS) of turbulent flow of generalized Newtonian (GN) fluids presented to date have shown significant discrepancy between experimental measurement and simulation. In addition to DNS, empirical correlations using different rheology models fitted to the same shear rheogram have also shown to give significantly different results. Important to note is that for turbulent flow predictions it is a common practice to use a shear rheogram which is measured at shear rates well below the values encountered in turbulent flows. This paper highlights the importance of obtaining high shear rate rheology in reducing these discrepancies. Further, it is shown that if high shear rate rheology is used in rheology characterisation, the choice of rheology model has little influence on the results. An important aside is that accurate prediction of laminar flow gives absolutely no confidence that a rheology model is acceptable in modelling the turbulent flow of the same fluid. From an analysis of instantaneous shear rates in the predicted turbulent flow field, the probability distribution of the non-dimensionlised shear rates in the near-wall region appears to collapse onto a universal curve. Based on this, we propose that the maximum shear rate required in rheology characterisation should be at least twice the shear rate corresponding to the mean wall shear stress.

1. Introduction

Many fluids in industrial applications and nature show non-Newtonian behaviour i.e. they do not show a uniform viscosity under isothermal conditions. Generalised Newtonian (GN) fluids are a class of non-Newtonian fluids for which the shear stress tensor $\boldsymbol{\tau}$ can be expressed as a product of a non-constant viscosity and the strain rate tensor:

$$\boldsymbol{\tau} = 2\mu(\dot{\gamma})\boldsymbol{S} \tag{1}$$

Here, $\dot{\gamma}$ is the second invariant of the strain rate tensor $\boldsymbol{S} = \frac{1}{2}[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T]$ determined as $\dot{\gamma} = \sqrt{(2\boldsymbol{S}:\boldsymbol{S})}$ and μ is a scalar viscosity usually called an effective or apparent viscosity. The GN assumption assumes an instantaneous response of the fluid to the applied shear stress and therefore, the viscosity of a GN fluid can be expressed as a function of shear rate $\dot{\gamma}$ as in Eq. 1. Note that the effective viscosity of a GN fluid can also depend on temperature, but we do not consider the effect of temperature in the current study. In practice, the effective or apparent viscosity of a GN fluid is determined by dividing the shear stress measured in a rheometer by the shear rate at which the stress is measured. These measurements are performed in a uni-directional flow in a rheometer. Fine particle suspensions, sewage sludges, molten lava, some polymer solutions, some bodily fluids and paints are examples of fluids that are well approximated by the GN assumption. Although the apparent viscosity of these fluids is often very high, industrially relevant flows can be turbulent at sufficiently high flow rates or in pipes with sufficiently large diameters. Despite their wide applications, there have been only a few studies dedicated to the fundamental understanding of turbulent flow of GN fluids, the majority of which have been experimental [1–6] with the primary objective often to derive a general correlation for the friction factor.

Unlike Newtonian fluids where the kinematic viscosity can be measured very accurately, non-Newtonian fluids are far more difficult to characterise. Despite this, the assumption of GN behaviour as a constitutive model appears to work well for a range of fluids. However, the constitutive equation relating the shear stress and shear rate is usually determined by fitting a particular mathematical rheology model to the experimental measured shear rheogram. There are many rheology models available for GN fluids [7,8], but the Herschel–Bulkley [9] and the Hallbom rheology models

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[10] have been found to represent the rheology of fluids such as mining and waste water slurries quite well [10–12]. The Herschel–Bulkley model defines the effective viscosity as:

$$\mu = \tau_y/\dot{\gamma} + K(\dot{\gamma})^{n-1} \quad (2)$$

Here, the yield stress τ_y, consistency K and flow index n are the model parameters. This model reduces to the power-law rheology model giving $\mu = K(\dot{\gamma})^{n-1}$ when $\tau_y = 0$ and the Bingham model $\mu = \tau_y/\dot{\gamma} + K$ when $n = 1$, both of which are commonly used in application. Unlike the Herschel–Bulkley model which has no theoretical basis [10], the Hallbom rheology model (Eq. 3) is derived by considering the behaviour of solid particles in homogeneous shear-thinning mineral suspensions and relates the viscosity and shear rate via the following equation:

$$\mu^k = (\tau_0/\dot{\gamma})^k + (\mu_\infty)^k \quad (3)$$

In this equation the model parameters are known as the yield stress τ_0, the infinite shear viscosity μ_∞ and the scaling factor k. The benefit of the Hallbom model in approximating (for example) a fine particle suspension is that as $\dot{\gamma} \to \infty$, the carrier fluid rheology is recovered, unlike the Herschel–Bulkley model in which the predicted viscosity drops below that of the carrier fluid. This is not physically possible.

Rheograms used for determining rheology model parameters in industrial application are typically measured over shear rates that would rarely exceed 500 s^{-1} (and often significantly less). This range is well below the shear rates that could be encountered in turbulent flow. Another way of determining the model parameters for the rheology models discussed here is via the use of analytical expressions that relate the bulk velocity (volumetric flow rate per unit cross-sectional area) and pressure gradient under laminar flow conditions to the model parameters [7,8]. These relationships can be (and often are) used for determining rheology parameters from the measured laminar flow curve (i.e. flow rate versus pressure drop), often in a small scale laboratory pipe loop. The shear rate range over which laminar flow is appropriate will depend on the fluid and pipe diameter. For laboratory experiments it is typically less than 300 s^{-1} and sometimes much smaller. It has been shown that constitutive equations based on different rheology models determined from the same laminar pipe data when used with theoretical or empirical correlations for determining the friction factor give barely distinguishable results in the laminar region as expected. However, the discrepancy in the turbulent regime can be as large as 50% [12,13]. Regardless, it is a common practice in hydraulic conveying to use such measurements.

Numerical techniques such as Reynolds Averaged Navier–Stokes(RANS), large eddy simulation (LES) and direct numerical simulations (DNS) require a constitutive equation for estimating viscosity. Although simulation of turbulent flow of GN fluids using these numerical techniques, particularly DNS, show encouraging outcomes [14–16], the most fundamental flow prediction (flow rate versus pressure drop, or equivalently friction factor) is usually in error. In DNS of pseudo–plastic fluids, Rudman *et al.* [14] found that for a given pressure gradient the bulk velocity predicted by DNS was 25% lower than the experimental value. Given that the same code at a similar resolution was able to predict the turbulent flow of a Newtonian fluid to within a few percent, this level of error is unacceptable. The discrepancy between DNS and experiments could be due to the following factors:

1. Inaccurate experimental measurements;
2. Poor choice of simulation method and parameters in terms of method accuracy, simulation resolution, computational domain length and time duration over which results are averaged;
3. The rheology measurements and/or data fitting;
4. The assumption of a GN rheology model.

Given that the experimental techniques used for the pipe flow measurements reported in Rudman *et al.* are standard and were validated against well characterised water data, experimental error is believed to be far smaller than the observed discrepancy. Thus, the first point is unlikely to be the cause. A spectral element-Fourier method which is exponentially convergent [17] was used in that study and domain length and mesh convergence were ensured, therefore, the second point is also unlikely to be the cause. In their simulations, Rudman *et al.* observed that the instantaneous, local shear rates spanned many orders of magnitude and were predicted to be significantly higher than those values used in the rheological characterisation. They suggested that the extrapolation of the shear rheogram for estimating viscosity beyond the range of shear rate where it was measured lead to the observed discrepancy between simulation and experiment. Thus the third point remains a possibility. The assumption that a GN model is appropriate is a difficult one to demonstrate conclusively. Although it remains a possibility that the GN assumption is not valid, we do not consider this as an alternative here. We agree with the argument in Rudman *et al.*, and later demonstrate, that the majority of the discrepancy arises due to poor rheology characterisation at high shear rates. It is worth noting that for turbulent flow predictions using empirical correlations (for example Dodge & Metzner [2] and Wilson & Thomas [18]), the importance of high shear rate rheology has also been advocated by other researchers [2,7,19]. Shook & Roco [19] suggest that for turbulent flow predictions, the shear rheogram used in rheology characterisation should be measured to shear rates at least as high as those corresponding to the mean wall shear stress τ_w.

The objectives of the present study are three-fold. First we aim to show that shear rheograms determined using traditional approaches such as laminar pipe flow curves or rheometry measured at low shear rates, when extrapolated to shear rates relevant to turbulent flows can deviate significantly from the actual rheology. By including the high shear rate rheology of the fluid in rheological characterisation, discrepancies between experiments and predictions using DNS or empirical turbulent flow correlations can be significantly reduced. Second, if an appropriate range of shear rates is considered in the rheology characterisation, the choice of the rheology model has a very small effect on turbulent flow predictions of DNS or empirical correlations. The third objective of this study is to define a criterion for the maximum shear rate (and shear stress) to use in rheology characterisation in order for DNS to produce good results. In the process of determining this we analyse the shear rate distribution in turbulent pipe flow field for the first time. The results suggest that in the near-wall region, the probability distribution of non-dimensionlised instantaneous shear rate collapses to a universal distribution for different models, fluids and Reynolds numbers. Based on this observation we propose that for turbulent flow predictions of shear-thinning fluids, the rheology characterisation should use the rheogram measured at least up to twice the mean wall shear stress.

2. Pipe flow measurements

The pipe flow test apparatus is shown schematically in Fig. 1. It comprises a 14 m pipe loop (≈300 diameters) with an internal diameter of 44.5 mm. A 400 litre agitated tank supplies a Warman $2 \times 1\frac{1}{2}$ AH variable speed pump for circulating fluids around the loop. The pressure gradients in both the upper and lower horizontal lines are measured using differential pressure (DP) cells spanning straight sections of pipe. The volumetric flow rate is monitored via a magnetic flow-meter. The rig instrumentation is data logged using a stand-alone LabVIEW application allowing the normal transport flow characteristics to be obtained in real time. In order to test the instrumentation a water-only flow curve was