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Homogeneous equilibrium model for geomechanical multi-material flow with compressible constituents

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ABSTRACT

Multi-material flow generally describes a situation where several distinct materials separated by sharp material interfaces undergo large deformations. In order to model such flow situations in the context of geomechanics and geotechnical engineering, a theoretical framework is presented which introduces a possible two-phase coupled saturated granular material behavior among the different materials. This is achieved by extending the technique of local volume averaging to a hierarchy of three spatial scales, based on a product of two indicator functions. A homogeneous equilibrium mixture model is subsequently derived for an example flow consisting of bulk solid, bulk fluid, and undrained granular material with compressible constituents. The closure relations are provided at the macroscale, including those describing granular behavior covering the full frictional-collisional flow regime and bulk material volume fraction evolution. The paper discusses the advantages and restrictions of the proposed mixture model and addresses its application and full-scale numerical implementation.

1. Introduction

Multi-material flow describes a situation where several pure, physically distinct materials (solids, liquids, gases) interact and one or more of these materials undergo large deformations—void, representing empty space or atmosphere, is generally considered as material. In contrast to multi-phase or multi-fluid flow [1–5], the main characteristics of multi-material flow are the evolution of large-scale material interfaces, including the generation of new free surfaces or the coalescence of existing surfaces, as well as the presence of material strength and compressibility. Moreover, in many situations mass transfer between the materials is of secondary interest, and momentum and pressure relaxation can be assumed infinitely fast, resulting in velocity and pressure fields common to all materials of the flow.

The notion of multi-material flow has emerged along with the development of efficient numerical simulation techniques to analyze detonation and impact problems, the dynamics of bubbles and droplets, material processing and manufacturing, or astrophysical events [6–8]. The most attractive approaches use Eulerian or arbitrary Lagrangian–Eulerian (ALE) descriptions allowing interfaces and free surfaces to flow through the computational mesh [9–14]. Mesh cells cut by interfaces (multi-material cells) necessarily arise which contain a heterogeneous mixture of two or more materials. In order to solve the same equations with the same numerical method, the heterogeneous mixture is represented as an effective single-phase material or homogenized mixture by using physically based mixing rules [15–17].

The research presented in this paper addresses multi-material flow situations encountered in geomechanics and geotechnical engineering. Examples are natural hazards like landslides [18,19], avalanches and debris flows [20–22], liquefaction-induced soil failure [23,24], and elementary installation processes like digging, injection, mixing, displacement, or penetration [25–29]. Schematic views are shown in Fig. 1. Besides the characteristics common to all multi-material flows, the aforementioned problems involve a complex coupled behavior of the dense grain-fluid mixture representing the soil or debris material as well as a hierarchy of distinct spatial scales [30]. While certain aspects of such geomechanical multi-material flows can be considered as well understood, a fully-fledged flow model that is able to predict a time history of the material states for arbitrary compositions and configurations of the mixture is yet missing.

The paper is concerned with the development of a macroscopic mechanical theory for compressible multi-material flow involving a hierarchy of three scales. We proceed from the premise not to

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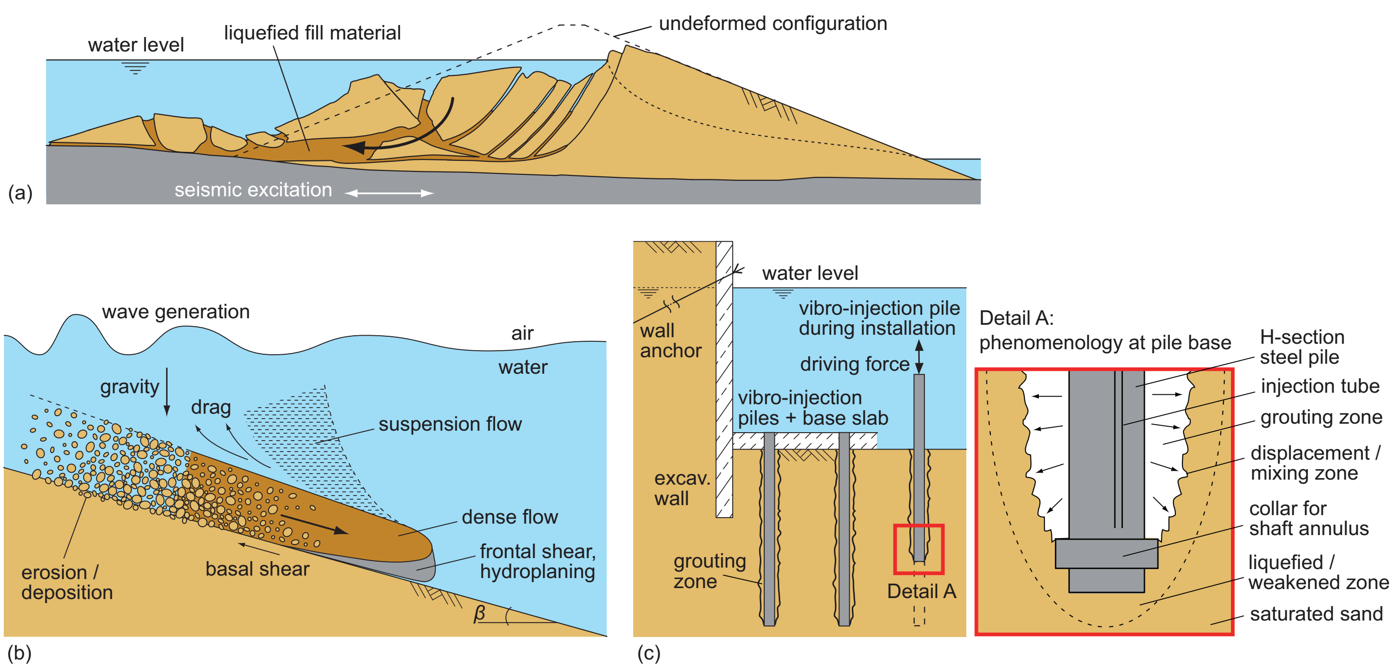

Fig. 1. Schematic of complex geomechanical multi-material flow situations. (a) Liquefaction-induced failure of an earth-fill dam under seismic excitation; in accordance with [23]. (b) Submarine landslide; in accordance with [18]. (c) Installation of vibro-injection piles to tie back the base slab of a deep excavation; in accordance with [28]. Reprint from [30, p. 189] with permission of Springer.

Nomenclature

Operators and special notations

$\cdot,\ :,\ \otimes$	single, double contraction, tensor product
$\cup,\ \cap,\ \setminus$	union, intersection, and difference of sets
$\langle\,\cdot\,\rangle$	spatial average
$\dot{(\cdot)}^{\alpha k}$	material time derivative of αk-field along $\mathbf{v}^{\alpha k}$
$(\,\cdot\,)^{[\alpha k]}$	limit value at αk-boundary
$\overset{\triangledown}{(\cdot)}$	Zaremba-Jaumann rate
$\|\,\cdot\,\|$	Frobenius norm
$\boldsymbol{\nabla}(\,\cdot\,)$	covariant derivative, gradient
div(·)	divergence
tr(·)	trace

Superscripts and subscripts

dev	deviator of a second-order tensor
dr	drained
e	elastic
f, fG, fk	fluid phase, in granular material, in k-material
fr	frictional (rate-independent) contribution
F	bulk fluid; F ≡ fF
G	fluid-saturated granular material
G′	related to effective stress in granular material
k	k-material; $k \in \{\mathrm{S,F,G}\} = \{1,\dots,M\}$
p	plastic
s, sG, sk	solid phase, in granular material, in k-material
S	bulk solid; S ≡ sS
T	transpose of a tensor
uj	unjacketed
vi	viscous (rate-dependent) contribution
α	α-phase; $\alpha \in \{\mathrm{s,f}\} = \{1,\dots,N\}$
αk	α-phase in k-material; $\alpha k \in \{\mathrm{S, F, sG, fG}\}$

Latin symbols

$\mathbf{b},\ \mathbf{b}^{\alpha k},\ \langle\mathbf{b}\rangle$	body force per unit mass
$\mathsf{c}^{\mathrm{S}},\ \mathsf{c}^{\mathrm{G}'}_{\mathrm{fr}}$	fourth-order material tangent tensor
$\mathbf{d}^k,\ \mathbf{d}^{\alpha k},\ \langle\mathbf{d}\rangle$	spatial rate of deformation
$\langle\mathbf{d}_k\rangle$	effective rate of deformation of k-material
dv	volume density
$\mathcal{D}$	modeling domain in Euclidian space $\mathbb{R}^3$
e^{G}	void ratio
f^k	k-material volume fractions
G	shear modulus
$\boldsymbol{h}^{\mathrm{S}},\ \boldsymbol{h}^{\mathrm{G}'}$	set of material state variables
$\mathbf{I}$	second-order unit tensor
J_2	von Mises stress invariant
$K^{\alpha k},\ K^k,\ \langle K\rangle$	bulk modulus
M	number of materials in the mixture
$\mathcal{M}^k$	k-material domain in $\mathcal{D}$
$n^k,\ n^{\mathrm{G}}$	fluid fraction, porosity
$\boldsymbol{n}_{\mathrm{I}}^{\alpha k}$	outward normals on interface
N	number of phases in the mixture
$p^{\alpha k},\ p^k,\ \langle p\rangle$	pressure
$p^{\mathrm{G}'}$	mean effective stress
$\mathcal{P}^{\alpha}$	α-phase domain in $\mathcal{D}$
$q,\ q^{\alpha k}$	generic spatial field
$\mathbf{s}^k,\ \mathbf{s}^{\alpha k},\ \langle\mathbf{s}\rangle$	extra stress
$\mathbf{v},\ \mathbf{v}^k,\ \mathbf{v}^{\alpha k},\ \langle\mathbf{v}\rangle$	spatial velocity
$\boldsymbol{v}_{\mathrm{I}},\ \boldsymbol{v}_{\mathrm{I}}^{\mathrm{m}}$	interface velocity
$V,\ V^k,\ V^{\alpha k}$	volume measures of $\mathcal{V},\ \mathcal{V}^k,\ \mathcal{V}^{\alpha k}$
$\mathcal{V}$	representative volume element (RVE)
$\mathcal{V}^k,\ \mathcal{V}^{\alpha k}$	portions of k, αk in $\mathcal{V}$
x	points in $\mathcal{S}$
y	yield condition

Greek symbols

$\boldsymbol{\Gamma}^{\alpha k}$	rate of momentum supply via $\partial\mathcal{V}^{\alpha k}$
ζ^{G}	Biot-Willis coefficient
$\Lambda^{\alpha k}$	rate of mass supply via $\partial\mathcal{V}^{\alpha k}$
$\mu^{\mathrm{fF}},\ \mu^{\mathrm{G}'}$	dynamic shear viscosity
$\pi^{\alpha k}$	volume fraction of α with respect to $\mathcal{V}^k$