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The effects of bond, shrinkage and creep on cracking resistance of steel and GFRP RC members

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A B S T R A C T

The analytical and numerical models for combining the effects of bond, shrinkage, creep and ageing of concrete on the stress-strain state and cracking resistance of reinforced or prestressed concrete elements in a period prior to loading are proposed.

A geometrical meaning of the coefficient of ageing is defined in terms of an average stress-strain approach. It is mathematically proven that this coefficient integrally establishes a continuous shape function of the actual concrete stress (or its increment) in a given period of time. This allows for substituting the rectangular stress diagram for the actual function of stress in time to attain the same areas of these diagrams and explicit predicting the stress-strain state in time by using algebraic equations for the inverse of Volterra integrals.

The analytical formulae for predicting the cracking moment of the cross-section accounting for bond, shrinkage, creep and ageing of concrete are also proposed. It is demonstrated that the intense development of the partially recoverable instantaneous and creep strains play the main role. The proposed formulae are applicable to the assessment or retrofitting of the existing structures, where the full contact interaction techniques are inappropriate.

1. Introduction

A considerable amount of the restrained shrinkage of concrete is, probably, the most common cause of cracking in reinforced concrete (RC) structures, even in a period before loading. Brown et al. [1] have reported that early age cracking induced by drying shrinkage affects more than 100,000 US bridge decks. Cracks can penetrate deep into the structure depth and shorten its service life. They are difficult to manage *in situ* and theoretical anticipation of these problems is important for design of massive liquid-retaining structures, nuclear waste storage facilities, waterfront engineering solutions, etc.

The development of the shrinkage-induced stress-strain state is a *displacement-imposed* phenomenon, governed essentially by the evolution of bond stresses at the contact interface of reinforcement and concrete. Acting along the span, they accumulate the shear contact forces exerted on concrete and reinforcement in the opposite directions, but being equilibrated at the contact interface. As the shrinkage displacement evolves, these forces counteract (restrain) the element's shortening in the same way as friction forces [2] and act on concrete as the tensile internal loads reducing the tensile strength. Generally, they are the slip-dependent unknowns resulting in a static indeterminability of RC element, emerging at the commencement of concrete curing and developing gradually in the process of drying.

For low bond stiffness, concrete shrinks almost freely without any induced tensile stress, because the shear contact forces are not generated. For stiffer bond, the longitudinal shear induces a partial interaction between concrete and reinforcement, and the tensile stress of concrete is transferred from the stiffer compressed bars via a shear force. A transfer monotonically decreases from the mid-span towards the element ends, while the cross-sections adjacent to the ends remain free from tension, since the maximal bond stresses are developed here. By introducing a full stress transfer option over the whole span, the shear contact forces are excluded from the equilibrium equations, because concrete and reinforcement stresses self-balance under the shrinkage displacement imposed.

A landmark paper on the solution of an interfacial bond problem for the lap joint of iron plates has been proposed by a Serbian scientist Arnovljević in 1909 [3]. He pioneered in deriving a second-order differential equation for the unknown shear contact force acting between the welded/riveted iron plates. Meanwhile, a generalized theory of the built-up bars, possessing a partial slip interaction at the interfaces, was developed by Rzhanitsyn [4] and used for a variety of analyses of

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statically determinate/indeterminate elements of metallic, wooden or multistory building structures, along with the solutions in the problems of stability and dynamical behavior [5]. For composite steel and concrete beams, a classic linear elastic partial-interaction theory was first given by Newmark et al. [7]. More complex analytical solutions for damage growth, cracking and failures at thin layers interfaces under monotonic temperature loading can be found in [6]. The considered monotonic temperature loading is relevant to modeling the gradual development of shrinkage strain.

There is a variety of nonlinear bond-slip constitutive laws proposed in the literature. However, any application of these laws generates a nonlinear differential equation of the second order, which is analytically irresolvable. To overcome this limitation, a *multi-linear* piecewise approximation of the actual bond-slip function is effective [8,9]. The actual bond stresses function can be also replaced by a piecewise rectangular stresses diagram in terms of their average values [10]. Meanwhile, a semi-analytical solution of second-order nonlinear differential equation has been demonstrated by Russo et al. [11], using the hypergeometric series function ${}^2F_1(\bullet)$.

The restrained shrinkage-induced internal forces generate the tensile creep strain of concrete. For the early-age concrete, this strain may reach substantially high values at the time of loading and results in considerable relaxation of the tensile stress of concrete. Moreover, the intense evolution of the modulus of elasticity in the period prior to loading generates the partially recoverable instantaneous elastic strains of concrete that affect the stress relief. Mathematically, a simulation of concrete creep phenomenon is a complicated task due to the stress rate and ageing-dependent effects.

Bažant [12] mathematically formulated an age-adjusted effective modulus method that has been found to be superior to other techniques [13]. It allows for achieving a theoretically-exact quasi-elastic solution of the time-dependent stress of concrete, if the coefficient of ageing is computed *in advance* through the inversion Volterra's integral for a given creep coefficient [12,14]. However, to find the unknown change of this stress, the *incremental* stress-strain relations are required. In this case, a sophisticated relaxation procedure, based on the gradually applied fictitious restraining actions to restore the equilibrium from the artificially-induced constant total strain of concrete in time, has been adopted for the cross-sectional analyses [15–19].

A mathematically simple and exact approach, based on the average stress-strain state definition aimed at fulfilling the Volterra's integral term, has been proposed by Balevičius [14]. The method does not require the introduction of the fictitious, incrementally-defined restraining actions dramatically simplifying the exact prediction of the time-dependent stress-strain state of RC uncracked elements. Moreover, the proposed expression for the ageing coefficient is of fundamental applicability and will be adopted hereinafter for evaluating the stress rate-dependent evolution of creep and instantaneous partially recoverable elastic strains. Due to intense ageing of concrete, a *strictly decreasing* function of this coefficient is required or the values of this function at the time of loading should be given for assessing the tensile stress relaxation for structural design.

Generally, the restrained-shrinkage induced tensile stress of concrete evolved prior to loading increases the level of cracking of the member. In this case, the interpolation-based moment-curvature approaches are highly sensitive to the magnitude of the cracking moment M_{cr} at the serviceability loads [18,20]. An effective stiffness model, based on constitutive modeling using the post-cracking stress-strain response of concrete in tension, gives a more complex solution for this limitation [21].

In [22], an attempt to improve the ACI 318 deflection prediction model with the reduction of M_{cr} by the fictitious restrained shrinkage-induced moment M_{sh} has been given. In fact, this concept is inappropriate because the tensile stress of concrete, resulting from the fictitious restrained shrinkage-induced axial force, is ignored. In particular, for the symmetrically reinforced element, there is no bending under the development of the shrinkage strain and $M_{sh} = 0$; thus, no decrease in the M_{cr} value will be obtained. However, due to the embedded bars, the restraining effect persists and induces the uniformly distributed tensile stress of concrete that reduces the tensile strength as well the cracking moment of the cross-section. On the contrary, when the tensile strength of concrete is reduced by accounting for the restrained shrinkage-induced tensile stress of concrete, these drawbacks are eliminated [18,20].

On the other hand, the design approaches are mostly based on a full contact interaction, where the slip at the interface is ignored. The conservatism embodied in full contact interaction techniques may be appropriate to the design of new structures, where the margins of safety are large, providing for an upper bound for tensile concrete strength and stiffness, and could be inappropriate to the assessment or retrofitting of the existing structures, where accurate methods of analysis are often in great demand by structural engineers.

The present work is aimed at achieving the analytical and numerical solution to combining the effects of bond, shrinkage, creep and ageing of concrete on the stress-strain state and cracking resistance of the regular, pre-tensioned or post-tensioned reinforced concrete elements in the period prior to loading.

2. The effect of bond and shrinkage on the cracking resistance of RC cross-section

2.1. The analytical model

Let us consider RC member of an arbitrary cross-section with y-axis of symmetry and the area of reinforcement A_s (Fig. 1a). The reinforcing bars, being in a continuous contact with the surface of the surrounding concrete, maintain the bond (tangential) stress $\tau_b(x)$, depending on the stiffness of contact (Fig. 1b). For the cracking resistance analysis, a simple linear elasto-brittle bond stress-slip law (Fig. 1b) can be adopted:

$$\begin{cases} \tau_b(s) = G_b s, \; |\tau_b| \leqslant \tau_{bu} \\ \tau_b(s) = 0, \text{ otherwise} \end{cases}, \tag{1}$$

where G_b is the slip modulus of bond (force units per cubic meter, Appendix A), $s \equiv s(x)$ is the slip displacement, $\tau_{bu} \approx 2\sqrt{f_c}$ is the bond strength [23], f_c is the compressive strength CEB-FIP (1990).

The bond stress accumulated over x and bar surrounding surface perimeter U gives a shear force (Fig. 1c)

$$T(x) = U \int_0^x \tau_b(x) dx, \tag{2}$$

which is transferred in the opposite directions for concrete and reinforcement, but is identical in the magnitude.

Each fibre of the cross-section (Fig. 1d) deforms under the applied loads and concrete shrinkage strain $\varepsilon_{shr}(t,t_s)$ developed in the period from the age of drying t_s to the age of loading t. The shortening due to shrinkage is supposed to be unrestrained by the supports (Fig. 1e), and $\varepsilon_{shr}(t,t_s)$ is uniformly distributed over the cross-section (Fig. 1c). Hence, $\varepsilon_{shr}(t,t_s)$ is treated as an *average* contraction strain over the depth. Thus, the problem couples the displacement and force-imposed phenomena and the total longitudinal displacement of concrete at any x can be expressed as

$$u_c(x) = u_{c\sigma}(x) - u_{shr}(t,x) = u_{c\sigma}(x) - \varepsilon_{shr}(t,t_s)x, \tag{3}$$

where $u_{c\sigma}(x)$ is the load-induced displacement, which is referred to be *positive* (Fig. 1c), if it resulted from the applied positive (tensile) force, $u_{shr}(t,x) = \varepsilon_{shr}(t,t_s)x$ is the shrinkage strain-induced displacement of concrete, which is referred to be negative as for the element shortening, i.e., it acts oppositely to the tensile force (Fig. 1c). The Eq. (3) can be expressed for the total strain of concrete as follows:

$$\varepsilon_c(x) = u_c'(x) = \varepsilon_{c\sigma}(x) - \varepsilon_{shr}(t,t_s), \tag{4}$$

where $\varepsilon_{c\sigma}(x)$ is the load-induced strain of concrete.