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Effect of non-affine motion on the centrifugal instability of circular Couette flow

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ABSTRACT

In the present work, the effect of non-affine motion of the constituents in a complex fluid system (say, a polymeric liquid) is theoretically investigated on the centrifugal instability of circular Couette flow. To achieve this goal, use was made of the linearized Phan-Thien/Tanner (LPTT) model thanks to its allowing non-affine deformation for the polymer strands in a temporary network of junctions through invoking a slip parameter. Knowing the basic-flow velocity and stress fields from the literature, they were subjected to infinitesimally-small, normal-mode perturbations and their time-evolution was monitored using a linear stability analysis for both the axisymmetric and non-axisymmetric modes. An eigenvalue problem was obtained this way which was solved numerically using the pseudo-spectral, Chebyshev-based, collocation method. Based on the results obtained in this work, it is concluded that the non-affine motion can have a stabilizing or destabilizing effect on circular Couette flow depending on the Weissenberg number and the sign/magnitude of the angular velocity ratio.

1. Introduction

Exact solution are indispensable tools in the field of fluid mechanics. This is because they always provide a better insight onto the physics of any particular problem at hand. They are also extremely useful for checking the performance of computer codes and algorithms. Unfortunately, exact solutions are quite rare in the field of fluid mechanics, and this is particularly so for any given non-Newtonian fluid. The chance of finding an exact solution becomes less probable when the degree of sophistication of the model is increased. For example, while for Giesekus fluids an exact solution does exist in plane stagnation-point flow [1], no such a solution could be found for the Phan-Thien/Tanner (PTT) fluids. This is rather unfortunate because the PTT model is one of the best rheological models when it comes to representing polymeric melts and concentrated polymer solutions. Having said this, it should be conceded that simplified forms of this robust rheological model render themselves to an exact solution in certain geometries [2–5]. One can particularly mention the exact solution found recently by Mirzazadeh et al. [5] in circular Couette flow for the linearized Phan-Thin/Tanner model (LPTT).

The problem with the exact solution found in Ref. 5 in circular Couette flow is that they have reported results for εWe^2 as large as 10, where ε is the extensional parameter of the LPTT model (see Section 2) and We is the Weissenberg number. Since in their work ε takes on values in the range of 0.01 to 0.1, one can conclude that in Ref. 5, the Weissenberg number has been allowed to assume values as large as 30. In practice, however, the purely-azimuthal solution found in Ref. 5 may have lost its stability long before such a high Weissenberg number can be reached. Obviously, having found an exact solution does not necessarily mean that the flow can materialize in the real world for arbitrary set of parameters. It is only through a (linear) stability analysis that a threshold can be obtained for the critical Weissenberg number below which the flow can exist in the real world. In the present work, we try to figure out the range of applicability of the exact solution reported in Ref. 5 for the linear LPTT model in circular Couette flow. Another objective of the work is to investigate the effect of the non-affine motion on the instability picture of circular Couette flow. The non-affine motion (which means internal slip between the constituents and the continuum) has been shown to affect the basic flow for the LPTT model [see Ref. 5] thus it is expected to affect the critical Weissenberg numbers. To the best of our knowledge, however, the effect of non-affine motion has not previously been investigated on the instability picture of circular Couette flow [6–15].

To reach its objectives, the work is organized as follows: knowing the basic flow from Ref. 5 for the LPTT model in circular Couette flow, we proceed with imposing infinitesimally-small, normal mode perturbations to the basic flow and invoke a linear,

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List of symbols

A	Coefficient matrices
B	Coefficient matrices
D	Deformation-rate tensor
m	Mode type
p	Pressure
R_1	Radius of the inner cylinder
R_2	Radius of the outer cylinder
Re	Reynolds number
V_{ss}	Basic flow velocity field
v_r	Radial velocity
v_θ	Azimuthal velocity
v_z	Axial velocity
t	Time
T_{ss}	stress field
We	Weissenberg number
α	Wavenumber
ε	Extensional parameter
ϕ	Radii ratio
$\dot{\gamma}$	Rate of shear or extension
η_0	Zero-shear viscosity
η	Apparent shear viscosity
η_E	Extensional viscosity
λ	Zero-shear relaxation time
Π_{ss}	Basic flow pressure field
ρ	Density
σ	Growth rate
τ	Deviatoric stress tensor
ξ	Slip (non-affine) parameter
ω	Angular velocity ratio
Ω_1	Angular velocity of the inner cylinder
Ω_2	Angular velocity of the outer cylinder

normal-mode, temporal stability analysis to determine the effect of non-affine motion on the critical Reynolds/Weissenberg numbers. The pseudo-spectral numerical method of solution is then described briefly before proceeding with presenting our new numerical results. The work is concluded by highlighting its major findings.

2. Governing equations

We consider confined flow between two infinitely-long concentric cylinders of radii R_1 and R_2 as shown in Fig. 1. Both cylinders are allowed to rotate with angular velocity Ω_1 and Ω_2, respectively–either in the same or in the opposite directions. We employ cylindrical co-ordinate system for our mathematical development (see Fig. 1). We also assume that the fluid is incompressible and takes no effect from the gravitational force while it is flowing in the gap between the two cylinders.

The equations governing the flow are the Cauchy equations of motion together with the continuity equation [16]:

$$\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \nabla.\underset{\sim}{\boldsymbol{\tau}} \tag{1}$$

$$\nabla.\mathbf{V} = 0 \tag{2}$$

where ρ is the density, $\mathbf{V}$ is the velocity vector, p is the isotropic pressure, and $\underset{\sim}{\boldsymbol{\tau}}$ is the extra stress tensor. We assume that the liquid of interest obeys the single-mode Phan-Thien/Tanner model (PTT) as its constitutive equation [17]. In its most general form, in this robust viscoelastic fluid model the stress tensor is related to the deformation field by the following relationship [18,19],

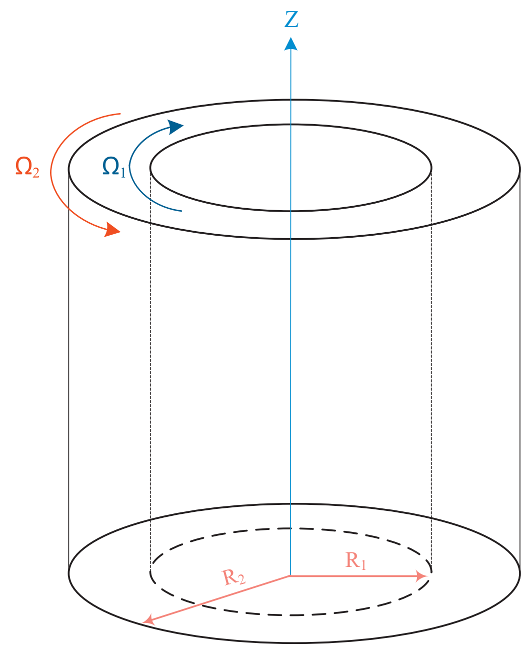

Fig. 1. Schematic showing the flow configuration.

$$\left(\exp\left(1+\varepsilon\frac{\lambda}{\eta_0}tr\,\underset{\sim}{\boldsymbol{\tau}}\right)\right)\underset{\sim}{\boldsymbol{\tau}} + \lambda\,\overset{\nabla}{\underset{\sim}{\boldsymbol{\tau}}} + \xi\,\lambda\left(\underset{\sim}{\boldsymbol{\tau}}.D + D.\underset{\sim}{\boldsymbol{\tau}}\right) = 2\eta_0 D \tag{3}$$

where ε is the extensional parameter, λ is the zero-shear relaxation time, η_0 is the zero-shear viscosity, ξ is the slip factor, and 2D is the rate-of-deformation tensor. In this equation, $\overset{\nabla}{\underset{\sim}{\boldsymbol{\tau}}}$ refers to the upper-convected derivative which is defined by:

$$\overset{\nabla}{\underset{\sim}{\boldsymbol{\tau}}} = \frac{D\underset{\sim}{\boldsymbol{\tau}}}{Dt} - (\underset{\sim}{\boldsymbol{\tau}}.\nabla\tilde{\mathbf{V}} + \nabla\tilde{\mathbf{V}}^T.\underset{\sim}{\boldsymbol{\tau}}), \tag{4}$$

where D/Dt is the material derivative. It is worth-mentioning that the ε-term in Eq. (3) allows the relaxation time of the model to become a non-linear function of the extra stress tensor. (In practice, this term accelerates the rate of stress decay at high stresses while it ensures that the stress vanishes when the strain approaches zero.) The ξ-term, on the other hand, alters the rate at which the stress builds up in the fluid, which is why it is allowed to be a function of the rate-of-deformation tensor and the stress itself. Now, before proceeding any further, we try to make these equations dimensionless by substituting:

$$r^* = \frac{r}{R_2}, \quad z^* = \frac{z}{R_2}, \quad t^* = \frac{t}{\rho(d^2/\eta_0)}, \quad v_i^* = \frac{v_i}{R_1\Omega_1},$$
$$\tau_{ij}^* = \frac{\tau_{ij}}{\eta_0\,(R_1\Omega_1/d)}, \quad p^* = \frac{p}{\eta_0(R_1\Omega_1/d)}, \tag{5}$$

where R_2 is the radius of the outer cylinder, and $d = R_2 - R_1$ is the gap spacing. It is worth-mentioning that the reason why $R_1\Omega_1$ has been used as the velocity scale is the fact that in rheometry the outer cylinder is often held fixed ($\Omega_2 = 0$)–even when it is not fixed, one often replaces the bob instead of the cup for altering the gap setting which is another reason why $R_2$is commonly used in place of R_1 as the length scale. Also, we would like to stress that although we have relied on $\rho d^2/\eta_0$ (the so-called *viscous* time scale) for making the *time* dimensionless, it is also possible to rely on $1/\Omega_1$ord$/R_1\Omega_1$(the so-called *inertial* time scale) for this purpose. As discussed by Drazin and Reid [30] they are both equally good as far as data reduction is concerned. In the present work, we have decided to rely on $\rho d^2/\eta_0$ simply because in our previous work