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Dynamic instability of variable angle tow composite plates with delamination

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ABSTRACT

In this paper, the dynamic instability of variable angle tow (VAT) plates with a single rectangular delamination is studied using an analytical model. The analytical model is derived from the principle of potential energy based on the classical laminated plate theory. Both global and local behavior of delaminated VAT plates in the dynamic instability analysis are accurately captured by the use of multiple Legendre polynomial series. The equations for the motion in dynamic instability problem are derived using Hamilton's principle. The dynamic instability regions are determined from the resulting Mathieu differential equations, which are solved using Bolotin's approach. To validate the proposed analytical model, both critical buckling loads and natural frequencies of delaminated VAT plates are evaluated and compared with FEM results. The influence of delamination on the buckling load, natural frequency and dynamic instability region (DIR) of delaminated VAT plates is examined by numerical examples. A parametric study is subsequently carried out to analyze the effect of linearly varying fibre orientation angles on the dynamic instability response of delaminated VAT plates. Finally, the mechanism of applying variable angle tows to improve the dynamic stability performance of delaminated composite plates is studied.

1. Introduction

Thin-walled structures like plates and shells that are used as primary components in aviation, automotive and civil industry are often subjected not only to static loads but also to dynamic loads. Structures under time-dependent (i.e. periodic) in-plane loads might lead to unacceptable vibration at a critical combination of excitation frequency and the amplitude of the axial load, namely *parametric resonance*. Understanding the characteristics of *parametric resonance* of composite structures under a dynamic loading is also of importance in practical designs. The advantages of applying variable angle tow (VAT) laminates that generally possess variable stiffness properties to improve structural performance have been clearly shown in previous works, in terms of buckling [1–5], postbuckling [6,7] and vibration [8,9]. However, dynamic stability problem of VAT composite structures subjected to in-plane periodic loads has received little attention in variable stiffness composites research community. Delamination caused by the impact of foreign objects (tool drops, runway debris, bird strikes etc) or manufacturing process, is one of the most common damage forms in composite laminates. The existence of delamination between plies reduces the structural stiffness, strength and load-carrying capacity, and thus may give rise to early instability or failure to composite structures. This paper presents an analytical study of dynamic instability performance of VAT composite plates with a rectangular delamination. The mechanism of varying fiber orientation angles (VAT) resulting in the improved dynamic stability of composite plates with delamination is thoroughly investigated.

A considerable amount of research has been done on the dynamic instability analysis of composite laminated plates without delamination. Bolotin [10] initially studied the dynamic instability of various elastic systems under periodic in-plane loadings. Afterwards, Birman [11] studied the dynamic instability of unsymmetrically laminated cross-ply plates under periodic biaxial loading. In his work, the principal dynamic instability region was determined analytically. Srinivasan and Chellapandi [12] used the finite strip method (FSM) based on classical laminated plate theory to perform the dynamic instability analysis for composite laminated plates under periodic in-plane load. Moorthy et al. [13] and Chattopadhyay and Radu [14] carried out a similar investigation using the finite element method based on the first-order and higher-order shear deformation theories to approximate the instability regions for moderately thick composite plates. In addition, Mond and Cederbaum [15] used the method of multiple scales to carry

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out the dynamic instability analysis of antisymmetric angle-ply and cross-ply laminated plates. Wang and Dawe [16] investigated the dynamic instability of composite laminated plate and prismatic plate structures. Sahu and Datta [17] performed the dynamic instability analysis of composite laminated plates subjected to non-uniform harmonically varying in-plane loading. Recently, Samukham et al. [18] investigated the dynamic instability of a VAT composite plate under periodic in-plane compressive loading using the finite element method. In their work, considerable benefits from steered fibers in improving the dynamic stability behavior of VAT composite plates without delaminations were demonstrated.

There also has been a significant amount of research on the dynamic response of composite laminated beam-plates or plates with delamination. Wang et al. [19] initially proposed the free model to perform the free vibration analysis for isotropic beam-plates with a through-the-width delamination by considering the coupling between flexural and axial vibrations of delaminated sublaminates. However, vibrational mode shapes obtained using the free model may be physically inadmissible for an off-midplane delamination. To address this problem, Mujumdar and Suryanarayan [20] developed a constrained model, in contrast to the free model proposed by Wang et al. [19], to study the natural vibration characteristics of laminated beam type structures. In their work, delaminated portions were constrained to have identical transverse deformations. Later, Shen and Grady [21] conducted an analytical and experimental investigation for the vibration response of a cantilever beam-plate with a through-the-width delamination. Lee [22] studied the vibration characteristics of delaminated beam-plates using finite element method based on the layerwise plate theory. Luo and Hanahud [23] developed an analytical model that can consistently explain the phenomena observed by experiments [21], to predict frequencies of delaminated beam-plates. In addition, Shu and Della [24] applied Euler-Bernoulli beam theory to investigate the free vibration of composite beams with two enveloping delaminations. In their work, both free and constrained models were taken into account in the formulation. Alnefaie [25] developed a three-dimensional finite element model to analyze the dynamic response of fibre-reinforced composite plates with internal delamination. More recently, Li and Qing [26] proposed a nonlinear spring-layer model based on the modified H-R (Hellinger-Reissner) variational principle to perform the free vibration analysis of composite laminated plates with delamination. Liu and Shu [27] adopted Euler-Bernoulli hypothesis to develop an analytical solution for the free vibration of exponential functionally graded beams with a single delamination.

However, there is not much literature available on the dynamic instability response of composite laminated plates, shells or other structures with delamination. Mohanty et al. [28] and Radu and Chattopadhyay [29] used the finite element method based on the first-order and higher-order shear deformation theories, respectively, to investigate the dynamic instability of composite laminated plates with delamination under uniform periodic in-plane loading. Yang and Fu [30] combined Rayleigh-Ritz method with classical shell theory to carry out the dynamic instability analysis for composite laminated cylindrical shells with delamination. Park and Lee [31] applied the higher-order shell theory of Sanders to study the dynamic instability of delaminated spherical shell structures subjected to periodic in-plane loadings. More recently, Noh and Lee [32] investigated the dynamic instability of delaminated composite skew plates under various periodic in-plane loadings. All of the above research works focused on the study of dynamic instability of constant stiffness laminated plates or shells with delaminations. Fazillati [33] recently reported a research work that applied a B-spline finite strip method to study the dynamic instability behavior of variable stiffness composite laminated plates with delamination. However, the non-uniform in-plane stress field was not considered in his model and analysis. A benign non-uniform in-plane stress redistribution given rise by variable stiffness properties had been recognized as the major driver for VAT composite structures to achieve improved performance [1–5]. The characteristics of dynamic instability of delaminated VAT composite plates therefore need to be further clarified.

In the present work, an improved analytical model based on the principle of potential energy and the Rayleigh-Ritz approach was developed to analyze the dynamic instability of VAT composite plates with a single delamination under periodically varying in-plane compressive loadings. In this delamination modelling, both global and local displacement shape functions constructed using the Legendre polynomials are introduced and the kinematic continuity conditions along the delamination edge are satisfied using the superposition method. The non-uniform in-plane stress distribution is determined prior to the dynamic instability analysis of delaminated VAT plates. The content of this paper is arranged as follows. In the next section, the concept of VAT laminates is introduced. Section 3 presents the basic formulae for the dynamic instability analysis of VAT composite plates with a single delamination, including the constitutive equation, the boundary conditions and the kinematic continuity conditions along the delamination edge. In Section 4, the modelling work for solving the in-plane stress and dynamic instability problems of delaminated VAT plates are presented. In Section 5, the accuracy and reliability of the proposed analytical model are validated by comparing with numerical results of buckling and vibration of delaminated composite plates with those obtained by FEM and prior results. The influence of delamination on the buckling, vibration and dynamic instability response of delaminated VAT plates is investigated by numerical examples. The mechanism of exploiting variable stiffness properties to improve the dynamic stability of composite laminated plates with a single delamination is also studied in detail. Finally, some conclusions are drawn in Section 6.

2. VAT laminates

The fibre orientation angle of the VAT composite plate varies continuously with spatial location over the entire plane of a ply. In this paper, the fibre orientation angle within a ply is assumed to vary linearly along the length of the plate, given by [34]

$$\theta(x) = \phi + \frac{2(T_1 - T_0)}{a}|x| + T_0 \tag{1}$$

where a is the length of the plate. T_0 is fibre angle at the center of the plate, that is, $x = 0$. And T_1 is fibre angle at the edges of the plate, that is, $x = \pm a/2$. ϕ is the angle of rotation of the fibre path [34]. The fibre orientation angle of a VAT ply is designated by $\phi\langle T_0, T_1\rangle$.

3. Formulation

3.1. Constitutive relation and strain-displacement relation

Consider a VAT composite plate of length a, width b and thickness h, with a single delamination, as shown in Fig. 1. The single delamination is located at mid-length, and the distance from the delamination interface to the top surface is h_1. The VAT composite plate is divided into three portions by the existing delamination interface, namely, an undelaminated portion, denoted by 0, and two delaminated portions, denoted by 1 and 2. The constitutive equations for the L^{th} portion of the

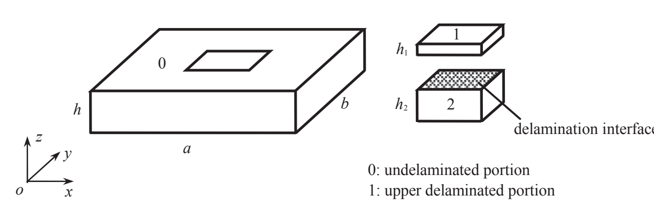

Fig. 1. A delaminated VAT plate is divided into three portions by the delamination interface.