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Contact line instability of gravity-driven flow of power-law fluids

Bin Hu, Sarah L. Kieweg*

Department of Mechanical Engineering, University of Kansas, 1530 W. 15th St., 3138 Learned Hall, Lawrence, KS 66045, United States

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ABSTRACT

The moving contact line of a thin fluid film can often corrugate into fingers, which is also known as a fingering instability. Although the fingering instability of Newtonian fluids has been studied extensively, there are few studies published on contact line fingering instability of non-Newtonian fluids. In particular, it is still unknown how shear-thinning rheological properties can affect the formation, growth, and shape of a contact line instability. Our previous study (Hu and Kieweg, 2012) showed a decreased capillary ridge formation for more shear-thinning fluids in a 2D model (i.e. 1D thin film spreading within the scope of lubrication theory). Those results motivated this study's hypothesis: more shear-thinning fluids should have suppressed finger growth and longer finger wavelength, and this should be evident in linear stability analysis (LSA) and 3D (i.e. 2D spreading) numerical simulations. In this study, we developed a LSA model for the gravity-driven flow of shear-thinning films, and carried out a parametric study to investigate the impact of shear-thinning on the growth rate of the emerging fingering pattern. A fully 3D model was also developed to compare and verify the LSA results using single perturbations, and to explore the result of multiple-mode, randomly imposed perturbations. Both the LSA and 3D numerical results confirmed that the contact line fingers grow faster for Newtonian fluids than the shear-thinning fluids on both vertical and inclined planes. In addition, both the LSA and 3D model indicated that the Newtonian fluids form fingers with shorter wavelengths than the shear-thinning fluids when the plane is inclined; no difference in the most unstable (i.e. emerging) wavelength was observed at vertical. This study also showed that the distance between emerging fingers was smaller on a vertical plane than on a less-inclined plane for shear-thinning fluids, as previously shown for Newtonian fluids. For the first time for shear-thinning fluids, these results connect trends in capillary ridge and contact line finger formation in 2D models, LSA, and 3D simulations. The results can provide us insights on how to optimize non-Newtonian fluid properties to minimize a fingering instability in many industrial and biological applications.

1. Introduction

Gravity-driven thin film flow with fingering instability is of interest in many fields, such as industry (paints [1], contact lens manufacture [2], and microchip fabrication [3]), nature (lava flow [4] and glacier flow [5]), and biomedical applications (microbicidal drug delivery [6,7], eye tears and substitutes [8]). In many of the applications, a uniform coating is desired with no dry spots. Thus, it is very important to understand the mechanics of the fingering instability at the moving contact line of a spreading thin film.

Numerous experimental and analytical/numerical studies have examined the dynamics of a gravity-driven contact line following the famous study of Huppert [9]. Schwartz [10] proved the contact line instability is controlled by surface tension effects. Troian et al. [11] carried out linear stability analyses (LSA) on thin film flow and derived the formulation under the limit of small wavenumber to show the capillary ridge was responsible for the instability. Bertozzi and Brenner [12] verified the LSA numerically and developed the transient model to investigate the transient growth of the fingering instability. Lin and Kondic [13] studied the instability of the thin film flowing down an inverted incline. These studies all assumed a constant flux configuration, however, in practical applications, constant-volume configuration is often needed. In our previous 2D study [6], we showed how the capillary ridge at the front of the flow evolves for a constant volume configuration. Espin and Kumar developed a 2D constant-volume model to study the thin film flow of colloidal suspensions, and showed both the particle concentration and the evaporation have a large impact on the front interface [14,15]. Gonzalez and Gomba developed a predictive model and integral method to study the linear stability of the constant volume flow [16,17]. All these studies provide a systematic approach to deal with the capillary ridge and contact line instability problem.

However, most of those previous studies were for Newtonian fluids. The fluids used in the above mentioned industrial and biomedical

* Corresponding author. Tel.: +1 7858643354; fax: +1 7858645254.
E-mail address: kieweg@ku.edu (S.L. Kieweg).

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applications usually exhibit non-Newtonian behavior, especially shear-thinning behavior. There are few published studies on contact line instability of non-Newtonian fluids. Balmforth et al. [18] studied the instability of Bingham fluids using LSA and showed the yield stress stabilized the contact line. Spaid and Homsy [19,20] used energy analysis for viscoelastic fluids to show elasticity has a stabilizing effect on the capillary ridge. It is still unknown how the shear-thinning behavior, for non-Newtonian fluids can affect the contact line instability. In our previous work [6], we completed a 2D analysis of shear-thinning fluids. Using travelling waves and numerical simulations of one-dimensional spreading, we found that increasing the shear-thinning behavior of polymer solutions decreased the capillary ridge height. This leads to the hypothesis for this study: that more shear-thinning fluids should have suppressed finger growth and longer finger wavelength, and that this should be evident in linear stability analysis and 3D numerical simulations. In summary, the relationship between the emergence and height of a capillary ridge in a 2D shear-thinning model has not previously been related to the linear stability analysis and 3D numerical model of the contact line instability. To solve this issue, we need to develop a contact line model of power-law fluids and identify the importance of different factors affecting fingering instability.

To verify the linear stability analysis for a Newtonian fluid, Kondic and Diez [21–25] numerically studied the 3D flow to simulate the fingering instability in the transverse direction. Lin et al. [26] studied 3D simulations for fluids on an inverted incline for unevenly distributed fluid viscosity. Those studies were also only for Newtonian fluids. Our research group has developed a 3D model for power-law fluids [27] and Ellis fluids [28] to study the spreading speed of a polymer solution and compare to experiments. However, those models did not incorporate the surface tension effect, and therefore cannot simulate the fingering instability.

The goals of this study were to: (a) in Section 2, develop a contact line model using LSA, and study how the shear-thinning effect would influence the finger growth, and (b) in Section 3, expand to 3D flow simulations with various perturbations to verify the LSA results.

2. Linear stability analysis

2.1. Methods for linear stability analysis

The fluid is described by power-law constitutive model: [29]

$$\tau_{ij} = m|II_{2D}|^{\frac{n-1}{2}}\left(2D_{ij}\right)$$

where $\tilde{\tilde{\tau}}$ is the stress tensor, m is the consistency of power-law fluid, $2\tilde{\tilde{D}} = (\nabla\tilde{v})^T + \nabla\tilde{v}$ is the shear rate tensor, $\tilde{v}$ is the velocity vector, and $II_{2D} = (1/2)[(tr2\tilde{\tilde{D}})^2 - tr(2\tilde{\tilde{D}})^2]$ is the second invariant of the shear rate tensor.

To describe the movement of the fluid's free surface flow down an incline, a wetting flow assumption and thin film lubrication approximation are commonly used. A non-dimensional partial differential equation (PDE) for the 3D flow (i.e. 2D spreading) of power-law fluids can be obtained for the height of the fluid as a function of space and time, $h(x,y,t)$. A similar detailed derivation was shown in Perazzo et al. [30] and our previous publications on power-law models [6,27]. The resulting non-dimensional thin film equation for a power-law fluid is:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left\{h^{\frac{1}{n}+2}\left\{\left[1 - D\frac{\partial h}{\partial x} + \frac{\partial(\Delta h)}{\partial x}\right]^2 + \left[D\frac{\partial h}{\partial y} - \frac{\partial(\Delta h)}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[1 - D\frac{\partial h}{\partial x} + \frac{\partial(\Delta h)}{\partial x}\right]\right\}$$
$$-\frac{\partial}{\partial y}\left\{h^{\frac{1}{n}+2}\left\{\left[1 - D\frac{\partial h}{\partial x} + \frac{\partial(\Delta h)}{\partial x}\right]^2 + \left[D\frac{\partial h}{\partial y} - \frac{\partial(\Delta h)}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[D\frac{\partial h}{\partial y} - \frac{\partial(\Delta h)}{\partial y}\right]\right\} = 0 \quad (1)$$

where n is the power-law index and $n < 1$ indicates shear-thinning fluids. The dimensionless parameter $D = cot\alpha(Ca)^{1/3}$ reflects the magnitude of the normal component of gravity force (e.g. $D = 0$ is vertical, $D = 1$ is inclined). The dimensionless parameter $Ca = \mu_0 U/\gamma$ is the power-law capillary number, α is the inclination angle, and γ is the surface tension coefficient. U is a characteristic velocity and μ_0 is a characteristic viscosity incorporating the power-law terms. These latter terms follow the dimensionless groups used for Newtonian fluids [12,21], and were further modified for the power-law variation as described in more detail in Appendix D of our previous study [6].

To conduct a linear stability analysis (LSA), we first determine a traveling wave solution. The method described here for traveling waves and LSA follows the general approach described in detail for Newtonian fluids by previous authors, e.g. in [12,21]. To find a traveling wave solution, we assume $h(x,y,t)$ is y-independent to reduce Eq. (1) to its 2D form. Then, we assume constant flux boundary conditions such that the fluid height is flat far from the moving front: $x \to -\infty,\ h \to 1$ and $x \to \infty, h \to b$, where $b \ll 1$ is the thickness of the precursor. This boundary condition leads to a traveling wave solution $h_0(x, t)$ in the x direction. Using a moving reference frame, $x^* = x - Ut$ traveling with velocity U, the following ODE for $h_0(x*, t)$ is obtained (dropping $*$ from here forward)

$$-Uh_0 + \left\{h_0^{\frac{1}{n}+2}\left\{[1 - D(h_0)_x + (h_0)_{xxx}]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}[1 - D(h_0)_x + (h_0)_{xxx}]\right\} = f \quad (2)$$

where the boundary conditions also result in the following expressions

$$U = \frac{1 - b^{\frac{1}{n}+2}}{1 - b},\ f = \frac{-b + b^{\frac{1}{n}+2}}{1 - b}$$

Eq. (2) was numerically solved (see Appendix C of [6]) for the traveling wave solution, which may form a capillary ridge. The presence and height of the ridge depends on many factors, such as D and the power-law index, n [6].

Next, we can use this traveling wave solution as the 'base' solution in the x direction. When we try to expand to the transverse y direction, we can simply assume the solution is in the form of a base state h_0 with a perturbation h_1, $h(x,y,t) = h_0(x) + \epsilon h_1(x,\ y,t)$, where h_0, h_1 are of $O(1)$ and $\epsilon \ll 1$, and substitute it into the thin film PDE (Eq. (1)). Only terms that are on the order of ϵ are kept in the resulting equation, and h_1 can be expressed as a Fourier transform using the superposition principle, $h_1(x,\ y,\ t) = \int_{-\infty}^{0} g(x,\ t)e^{iqy}dq$, where q is the wavenumber. We apply the Taylor series to expand the power terms in Eq. (1). The Taylor approximation is kept in the same order of ϵ. We also use the traveling wave solution of Eq. (2) to substitute the higher order terms. After simplification, we can obtain a PDE for $g(x, t)$:

$$\frac{\partial g}{\partial t} + \frac{\partial}{\partial x}\left\{\left\{\left[\frac{(Uh_0 + f)}{h_0^{\frac{1}{n}+2}}\right]^{2n}\right\}^{\frac{1}{2n}-\frac{1}{2}} \times \left\{\left(\frac{1}{n} + 2\right)h_0^{\frac{1}{n}+1}\left\{\left[\frac{(Uh_0 + f)}{h_0^{\frac{1}{n}+2}}\right]^2\right\}^{\frac{n}{2}-\frac{1}{2}} \times \left[\frac{(Uh_0 + f)}{h_0^{\frac{1}{n}+2}}\right]g + \frac{1}{n}h_0^{\frac{1}{n}+2}[(-q^2 - D)g_x + g_{xxx}]\right\}\right\}$$