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Design of blended/tapered multilayered structures subjected to buckling constraints

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ABSTRACT

A general analytical procedure is proposed to design tapered laminated composite structures. It is based on: 1) the introduction of two variables representing a laminate configuration, 2) the analytical determination of the lower bound of the number of layers in each of the segments constituting the construction, 3) the definition of the reference sublaminate which configuration is transferred to all segments/parts of the construction, 4) the gradual building of configurations for symmetric balanced laminates having odd or even number of plies. The solution is verified by the comparison with results existing in the literature for both inner and outer tapers. A multi-panel composite structure is considered to demonstrate the applicability of the proposed method. As usual, in a such class of problems, the whole construction is divided into segments/panels. The individual segments are subjected to the simultaneous action of in-plane, tensile or compressive and shear loads. The present results demonstrate the simplicity and effectiveness of the method and non-uniqueness of solutions. The paper intends to clarify the physical sense of the discussed problem.

1. Introduction

Tapered/blended laminated structures have became the standard parts of modern engineering constructions. The use of thin rectangular plates with thickness that varies in the directions parallel to the two sides can help the designer to reduce the weight of the structure. For cases where reduction of weight is of high importance, such as space structures, this type of plate is the best choice. The buckling load for these plates is or can be a key factor in design considerations. In particular, buckling of stepped plates has attracted much more attention in the past few decades. This type of plate is used extensively because of its high strength to weight ratio. A variety of theoretical approaches have been formulated for this class of problems. These approaches may be applied to study varying thickness plates where the plate thickness is allowed to vary either as piecewise constant step functions, a linear function and piecewise linear functions, or as a non-linear function.

The blended/tapered composite structures, formed by dropping off the plies along mid-plane either internally or externally, are being widely used in turbine blades, helicopter yokes and robot arms due to their variable stiffness along various directions.

The behaviour of tapered composite laminates under static and fatigue loading has been widely addressed in the literature – see e.g. the review papers [1,2]. In the literature survey on tapered/blended composite structures the authors bring out three major categories of taper configurations: internal taper, mid-plane taper and external taper. Fig. 1 demonstrates the classical internal taper configuration.

A discussion of buckling behaviour of tapered laminated beams or plates have also received much attention from researchers. A large number of investigators have conducted the research on this subject [3–11]. The buckling analysis of tapered/blended structures is always associated with the optimization analysis of stacking sequences in all of adjacent segments in the construction considered. It is worth to point out that the allowed ply orientations are reduced to a discrete set of angles such as: {0, ± 45, 90} or {0, ± 15, ± 30, ± 45, ± 60, ± 75, 90}. In the composite construction thickness variations are obtained by drop ping plies at specific segments (regions). However, it is necessary to emphasize that the optimization analysis is always reduced to the use of the “black box” called as genetic algorithms (GA) or evolutionary algorithms (EA). It is well-known that the above algorithms allows us to obtain better solutions than initial ones but their location with respect to the global optimum is unknown.

It is well-known that no search algorithm is superior to any other algorithm on average across all possible problems. A consequence of this is that if an algorithm, say genetic algorithm, performs better than random search on a class of problems, that same algorithm will perform worse on a different class of problems [12]. From this one might draw the erroneous conclusion that there is no point in trying to find better algorithms. However, since we typically are not interested in all possible problems, this is not a case.

The objective of the present work is to fill this gap between the approximated optima obtained with the use of GA or EA and the global

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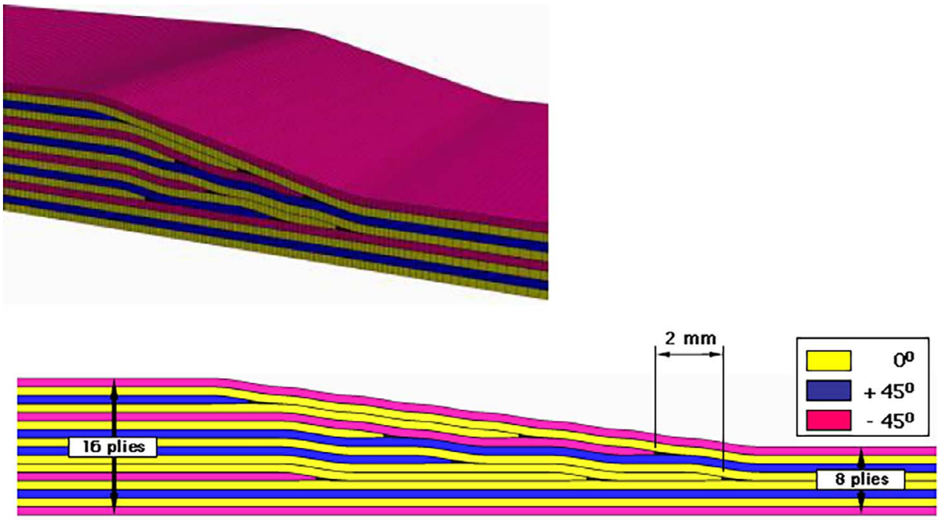

Fig. 1. Schematic illustration of the internal taper configuration.

optima by providing exact, analytical solutions to the buckling of tapered multilayered laminated simply supported rectangular plates. It is reached by the implementation of new types of design variables Muc [13,14]. The analytical method of solution allows to include also arbitrary constraints arising from various manufacturing requirements.

For aerospace complex structures made of a large number of panels it is customary and commonly accepted to divide constructions into many segments and design each of them independently [9,15–17]. Since the load distribution and wing stiffness are membrane dominated ([9] – Fig. 1, [16]) the panel local load approximations are computed numerically using the membrane FE. The details of the procedure are described e.g. by Ragon et al. [16], Meddaikar et al. [18]. The membrane loads may vary between segments so that the optimal thicknesses of them may be also different. To ensure continuity and manufacturability between adjacent composite segments the concept of blending was introduced by Kristinsdottir et al. [19] and then generalized by van Campen et al. [20]. In the present approach the load redistribution between panels due to the thickness variations in the local panel design is not taken into considerations. It is assumed that the load distribution at the panel level is fixed.

This paper is organized as follows. Theoretical formulations are presented in Section 2. The next section presents the formulation of the optimization problem, Section 4 is devoted to results and discussions. Three types of problems including buckling optimization are considered, i.e. buckling of a single monolithic plate, buckling of two segment panels and buckling of multi-segment blended panels. The paper ends with conclusions.

2. Buckling of multilayered bi-axially compressed plates

For structural modeling of the plate subjected to the bi-axial compression it is assumed that the coordinate system origin, is located at the plate corner on the mid-plane – Fig. 2. It is also assumed that the plate is made of N layers where each of plies has the identical thickness t/N (t is the total thickness of the panel). The plate is enforced to be symmetric about its mid-plane, requiring that only a half of the layers (i.e. $N/2$) be designed. In addition, the plate is also required to have a balanced stacking sequence.

Under the above assumptions for a simply supported plates subjected to the bi-axial compression the plate buckles when the parameter λ reaches the critical value expressed in the following way:

$$\lambda = \frac{(m\pi/L_x)^2}{N_x(1+k\beta_m^2)}[D_{11} + 2(D_{12}+2D_{66})\beta_m^2 + D_{22}\beta_m^4],\quad \beta_m = (nL_x)/(mL_y),\quad k = N_y/N_x, \tag{1}$$

where L_x, L_y are geometrical plate dimensions, and m, n are numbers of half-waves in two perpendicular directions corresponding to the plate co-ordinate system, and N_x, N_y are axial compressive forces per unit length in the longitudinal x direction and in the transverse y direction, respectively (Fig. 2). D_{11}, D_{12}, D_{66} and D_{22} are classical bending stiffness terms of the laminate. For the symmetric laminates the relation (1) can be rewritten in the more convenient form:

$$\lambda = gN^3(a_1 + a_2z_1 + a_3z_2), g = \frac{(m\pi/L_x)^2 t_L^3}{12N_x(1+k\beta_m^2)} \tag{2}$$

$$a_1 = U_1{-}U_3 + 2(U_1+3U_3)\beta_m^2 + (U_1{-}U_3)\beta_m^4, a_2 = U_2(1{-}\beta_m^4), a_3 = 2U_3(1{-}6\beta_m^2+\beta_m^4),$$

$$z_1 = \left(\frac{2}{N}\right)^3 \sum_{i=1}^{i=N/2}[3i(i{-}1)+1]\cos(2\theta_i), z_2 = \left(\frac{2}{N}\right)^3 \sum_{i=1}^{i=N/2}[3i(i{-}1)+1]\cos^2(2\theta_i)$$

$$U_1 = \frac{1}{8}(3Q_{11}+3Q_{22}+2Q_{12}+4Q_{66}), U_2 = \frac{1}{2}(Q_{11}{-}Q_{22}), U_3 = \frac{1}{8}(Q_{11}+Q_{22}{-}2Q_{12}{-}4Q_{66}),$$

$$U_4 = \frac{1}{8}(Q_{11}+Q_{22}+6Q_{12}{-}4Q_{66}), U_5 = \frac{1}{2}(U_1{-}U_4)$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, Q_{66} = G_{12}.$$

where θ_i denotes an angle of the fibre orientation of the ith layer, measured from the plate x-y axes to the 1–2 material axes and t_L is the thickness of the individual ply. In the material coordinate system 1–2 E_1, E_2, G_{12}, ν_{12} denote the Young moduli in the directions 1, 2, the Kirchhoff modulus and the Poisson ratio, respectively.

In the design space (z_1, z_2) the slope of the contours of the constant λ is characterized by the value of the constant a_2. It is positive for $a_2 > 0$ when $\beta_m < 1$, and negative when $\beta_m > 1$.

For rectangular plates made of plies having continuous angle-ply fibre orientations ($\pm\theta$) the optimal orientations that maximize buckling loads can be easily derived from the following relation (see Muc [21,22]):

$$z_1 = \cos(2\theta_{opt}) = -\frac{a_2}{2a_3} \text{ or } \theta_{opt} = 0 \text{ or } \theta_{opt} = 90, z_2 = z_1^2 \tag{3}$$

Let us note that for plates the optimal fibre orientations are the function of the geometrical ratio L_x/L_y only (unimodal solutions). As it is well-known for the identical value of the parameter k, the position of the maximal buckling load with respect to fibre orientations vary with the buckling mode (m, n) and the geometrical ratio (L_x/L_y). It is obvious