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Propulsion of axisymmetric swimmers in viscoelastic liquids by means of torsional oscillations

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ABSTRACT

Unconventional methods are needed in order to propel swimming objects in viscoelastic liquids. The paper deals with a locomotion principle based on a cyclic action which utilizes the elasticity of the fluid. The theoretical model considers any incompressible simple fluid of arbitrarily long memory and any axisymmetric swimmer of arbitrary profile which performs torsional oscillations of small amplitude. Perceiving the flow as unsteady perturbation of the rest state, an asymptotic analysis is developed, particularly with regard to the time-averaged speed of the swimmer in second-order approximation. In doing so, also inertia effects are considered in addition to the memory and the normal stress effects. A generalized reciprocal theorem including fluid elasticity proves to be extremely useful. It enables calculating the driving force on the swimmer without solving the second-order flow problem. General results are illustrated by means of a spherical swimmer. The analytical findings clearly show the influence of different process parameters including certain frequency dependent constitutive parameters on the driving force, on the swimming speed and on the secondary flow field.

1. Introduction

Viscous flows around small particles are often characterized by low Reynolds number, so that the fluid inertia may be neglected in a theoretical analysis. One speaks about creeping flows. If the fluid is Newtonian, they have particular properties because the dynamic equations are linear and time is a parameter only. As a result, any unsteady creeping Newtonian flow may be understood as sequence of quasi-steady states and it is time-reversible [1]. Moreover, flow fields being subject to kinematic boundary conditions are independent of the viscosity of the fluid.

Unconventional methods are needed in order to propel an object which shall swim under those conditions. Due to the time-reversibility, cyclic actions with a single degree of freedom, for instance the application of a rigid flapper, do not work. In fact, a self-propelled swimmer must execute a loop in a multi-dimensional configuration space [2]. Microorganisms manage it by deforming the body shape, particularly by the use of cilia and flagella. We refer to a review on the biological facts and on the fluid mechanical modelling [3]. Prominent models are so-called squirmers being (hypothetically) able to distort their surface, flexible cylindrical tails as mechanical wave guides and rigid helical bodies which rotate at constant speed.

Most publications concerning self-propulsion in Newtonian fluids under the condition of creeping flow deal with the propelling force produced by those mechanisms and with the resulting speed of otherwise force-free swimmers. However, these studies are not especially relevant in our context because we will analyse an alternative principle of locomotion which utilizes the elasticity of the fluid and which does not work in a Newtonian fluid because of the time-reversibility. So, we mention here only two recent papers. The one is dedicated to rotating helical bodies as they swim through a Newtonian fluid [4]. Assuming a nearly circular cross-section, closed form analytical predictions have been derived for the swimming speed (which is independent of the fluid viscosity) and for the torque (proportional to the viscosity). Helical bodies with more complex cross-sections were included by numerical simulations. The other paper deals with the hydrodynamic performance of different cilia beating patterns reconstructed from experimental data [5]. The swimming speed, certain internal moments generated by the cilia and the swimming efficiency were used as performance measures. Both publications contain extended lists of references which allow interested readers to trace back most of the original work on swimming in linear-viscous fluids.

It should be mentioned that a helical body which propels a swimmer forward could also be used as rotor of a screw pump which

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pushes the liquid backward. Both processes are based on the same functional principle. However, the swimmer moves within the laboratory frame whereas the pump is normally arranged at a fixed position. A similar analogy exists between self-propelling and pumping by means of travelling waves along a flexible body surface which will help us to understand theoretical findings.

With different applications in mind, studies on self-propulsion are expanded since a couple of years assuming that the fluid has more complex rheological properties. Both in some biological and some technical systems, small particles “live” in polymeric liquids. It is therefore important to explore how microorganisms swim in such a viscoelastic environment, and how artificial robots should be actuated in order to propel them. Of course, retaining established strategies, effects of the viscoelasticity on the swimming speed can be expected. However, existing reports caused some controversy as to whether the fluid elasticity enhances or hinders locomotion.

A simple model of a self-propelled swimmer consists of a flexible sheet transmitting a sinusoidal wave train of small amplitude. In case of a Newtonian fluid, the swimming speed U_N is proportional to the wave speed and is independent of the viscosity. Assuming the constitutive equations of an Oldroyd-B fluid, of a Johnson–Segalman fluid or of a Giesekus fluid, a second-order theory with respect to the wave amplitude leads in each case to the normalized swimming velocity $U/U_N = (1+\lambda_1\lambda_2\omega^2)/(1+{\lambda_1}^2\omega^2)$, where λ_1 is the relaxation time of the fluid, λ_2 is the retardation time and ω is the frequency of the wave [6]. Owing to the inequality $\lambda_2 < \lambda_1$, the swimming speed U is always smaller than the Newtonian value. Exactly the same formula results from an asymptotic analysis concerning the locomotion of an infinitely long circular cylinder due to a small-amplitude travelling wave in an Oldroyd-B fluid [7]. These findings become comprehensible considering the above mentioned analogy between force-free swimming and free pumping (without overall pressure difference). Already three decades before, it has been shown that, under the condition of free pumping, the volume flux Q of a creeping plane flow due to a travelling wave train, normalized by the Newtonian value Q_N, is influenced by the real part $\eta'(\omega)$ of the complex viscosity and by the zero-shear viscosity η_0 of the fluid, $Q/Q_N = \eta'(\omega)/\eta_0$ [8]. The formula is true also for axisymmetric peristaltic pumping [9]. Accordingly, linear-viscoelastic fluid properties control the pumping throughput and, analogously, the swimming speed in a second-order approximation. Normal stress differences exert influence only at higher approximation order. Consequently, constitutive models with the same complex viscosity like those of Oldroyd, Johnson–Segalman and Giesekus give rise to the same results. These findings suggest that fluid elasticity impedes locomotion of flexible swimmers. In contrast, there are indications of an increase of the swimming speed due to nonlinear viscoelastic response. Numerical simulations of large-amplitude flagellar beating in an Oldroyd-B fluid revealed enlargement factors up to 1.25 at Deborah numbers near one [10]. Finally, we point to a numerical study concerning the locomotion of a spherical swimmer within a nonlinear Giesekus fluid caused by the first two squirmer modes [11]. Accordingly, the steady flow around a force-free sphere was simulated under a prescribed surface velocity in polar direction corresponding to $v_\vartheta = (B_1 + B_2\cos\vartheta)\sin\vartheta$. The sign of the ratio B_2/B_1 controls whether the swimmer gets impetus from its front part or whether the thrust comes from the rare part. Numerical results regarding Weissenberg numbers up to 10 show that the swimming speed remains systematically below the Newtonian value. Major differences are found in the stress field.

In summary, we see that the fluid elasticity modifies the speed of a free swimmer, but does not produce great enhancements using the propulsion strategies which work already in case of a Newtonian fluid. Contrary to these studies, we discuss in the following an alternative principle of purely elastic propulsion. In doing so, we are encouraged by a recent experimental study which proves that rigid objects can move in a viscoelastic fluid under time-reversal stimulations [12]. It should be mentioned that in all above-quoted papers concerning propulsion in viscoelastic fluids, the influence of fluid inertia is completely neglected. In view of microorganisms within their natural environment, the assumption of creeping flow may be appropriate. However, concerning artificial robots with much larger length scales, it is not certain a priori whether unsteady and convective inertia forces are small compared to viscous and nonlinear elastic forces, respectively. Thus, we consider these effects in the analysis. Creeping flow situations are included as special cases.

The mechanism which we will analyse shall be explained roughly, supported by a few elementary formulas. Imagine the unsteady shear flow of a Newtonian fluid (density ρ, viscosity μ) near a flat plate that oscillates harmonically with frequency ω and displacement amplitude εa in its plane (in x-direction). An exact solution of the Navier–Stokes equations is known which indicates that the velocity field $v_x(y, t)$ of the unidirectional fluid flow can be represented as follows [13]:

$$v_x(y,t) = \varepsilon a Re\left\{ \exp\left(-\left(\frac{i\rho\omega}{\mu}\right)^{1/2} y + i\omega t\right)\right\}. \quad (1)$$

$Re\{\dots\}$ indicates the real part of the complex expression within the brace. The corresponding shear stress $\tau_{xy}(y, t)$ oscillates in phase with the shear rate $\partial v_x(y, t)/\partial y$ and the normal stresses σ_{xx}, σ_{yy}, σ_{zz} are equal and even spatially constant.

A lot changes if the fluid is viscoelastic with more complicated constitutive properties. Assuming that ε is small, the theory of linear viscoelasticity is appropriate in a first approximation. In doing so, the following correspondence principle proves to be most helpful: given any solution of the linearized Navier–Stokes equations under harmonically oscillating kinematic boundary conditions, then, in complex notation, there is a corresponding solution of the linear-viscoelastic problem under the same boundary conditions with the complex viscosity $\eta^*(\omega)$ used instead of the Newtonian viscosity. Thus, replacing μ by $\eta^*(\omega)$, Eq. (1) represents also the first-order velocity field of a viscoelastic fluid. Since the viscosity is complex now, shear stress and shear rate oscillate no longer in phase. Moreover and more important in the present context, unsteady normal stress differences arise in the shear flow under consideration, the time-averaged values of which are especially relevant. Restricting to a second-order approximation with respect to ε, we may assume

$$\langle \sigma_{xx}(y,t) - \sigma_{yy}(y,t)\rangle = 2\kappa(\omega)\left\langle \left(\frac{\partial v_x(y,t)}{\partial y}\right)^2\right\rangle \quad (2)$$

with positive normal stress coefficient $\kappa(\omega)$. We ignore here for a moment the second normal stress difference $\langle\sigma_{yy} - \sigma_{zz}\rangle$, which is often relatively small in real liquids, especially in polymer fluids. The expression in Eq. (2) may be understood as time-averaged extra-stress in flow direction in addition to the constant pressure. It does not produce any mechanical effect on the plate since the streamlines are rectilineal. But things are different if the streamlines are curved.

In the paper, we study the situation sketched in Fig. 1. An axisymmetric body of dimension a is immersed into an incompressible viscoelastic fluid at rest and performs torsional oscillations of frequency ω and of small angular amplitude ε. Under no-slip conditions at the body surface Γ, the fluid is forced to oscillate in circumferential direction. Consequently, the streamlines are circular in linear approximation, and the mentioned extra-stress is a normal stress in φ-direction. Since it decreases exponentially with increasing distance from the body, it occurs potentially only within a boundary layer near the surface. So, the time-averaged extra-stress field takes effect like an inhomogeneously stretched elastic membrane enclosing the axisymmetric body. It generates a second-order driving force acting on the body in axial direction for symmetry reasons. We expect a relation of the kind $\langle F_z\rangle = \varepsilon^2\kappa\omega^2 a^2 C_\kappa(S, V)$ where the non-dimensional factor C_κ is