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Interfacial instability in two-layer Couette–Poiseuille flow of viscoelastic fluids

Paresh Chokshi*, Supriya Gupta, Sheshnath Yadav, Ankit Agrawal

Department of Chemical Engineering, Indian Institute of Technology Delhi, New Delhi 110016, India

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The temporal linear stability analysis of shear flow of two-layer viscoelastic fluids, both described using the upper convected Maxwell (UCM) model, is carried out. The plane Couette flow under creeping flow conditions both in the absence and presence of pressure gradient is examined to study the role of unmatched elasticities in the interfacial instability. As the most critical disturbance is known to be of finite wavelength of the order of channel width, the numerical analysis examines the entire range of disturbance wavenumbers and the stability map is constructed in the parametric space of fluid Weissenberg numbers to indicate the region of stable interface. With a focus on optical fiber coating process, the analysis investigates the role of pressure gradient imposed on the plate-driven flow. Both the adverse and favorable pressure gradients are analyzed to identify the region for stable coating flow. In the presence of pressure gradient, the region of stable interface broadens when the high elasticity fluid occupies the region of low shear rate, which is the bottom (top) layer for the adverse (favorable) pressure gradient. For the two fluids of unmatched rheology, in addition to elastic instability, the role of viscosity stratification leading to a jump in the shear rate at interface is also examined. The adverse pressure gradient has a stabilizing effect when more viscous fluid is more elastic, whereas the favorable pressure gradient tends to stabilize the interface when the less viscous fluid has higher elasticity than the more viscous fluid.

1. Introduction

The two-layer shear flow of viscoelastic fluids is commonly encountered in polymer processing applications, like co-extrusion of polymer melt and two-layer coating of a glass fiber with polymeric resins. In later application, the primary coating resin is first cured before applying the secondary coating. If the first coating resin is only partially cured when the second layer of polymeric coating is dragged on to the fiber, the flow in the secondary coating cup (known as wet-on-wet coating) resembles the shear flow of two-layer viscoelastic fluids caused by the drag force on the glass fiber [1]. For the shear flow of two or more layered fluids, the interface has tendency to become unstable, which manifests in the form of sustained traveling wave oscillations at the interface. The unstable interface during the flow creates defects in the final product and hence, the instability is undesirable. The stability of the interface strongly depends upon the properties of the fluids, in addition to the imposed shear rate. Clearly, when the physical properties of the two fluids are matched, the two-layer flow resembles a single fluid flow and the interface is expected to be stable. The stratification of properties, like density, viscosity and elasticity, is responsible for the interfacial instability. The relevant prior work on the instability in stratified fluid flow is briefly reviewed below. A thorough review of the interfacial instability in two-layer flow with a detailed discussion on instability mechanism is presented in [2].

The plane Couette flow of two-layer Newtonian fluids with equal densities but differing viscosities is unstable to long wavelength disturbances. Yih [3], with the help of longwave asymptotic analysis for plane Couette flow and plane Poiseuille flow, found that the interface can become unstable solely due to viscosity stratification even when the Reynolds number is very small. Yiantsios and Higgins [4] further confirmed the instability due to stratified viscosity and found that the instability is suppressed by confining the less viscous fluid in a thinner layer. Importantly, this long wave instability occurs in the presence of inertia, requiring the Reynolds number to be non-zero, however, vanishingly small. Even in the absence of long wave instability, the flow can still become unstable to disturbances of short wavelength [5]. This instability, in which the disturbance is localized at the interface, is attributed to jump in the shear rate across the interface. The presence of interfacial tension is known to stabilize the shortwave instability. The shortwave instability due to unmatched viscosity also occurs for small but non-zero Reynolds number, necessitating convection of disturbance vorticities leading to the growth

* Corresponding author. Tel.: +91 11 2659 1053; fax: +91 11 2658 1120.
E-mail address: paresh@chemical.iitd.ac.in (P. Chokshi).

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of the perturbed interface [2]. Thus, the two-layer flow of Newtonian fluids with different viscosities is stable under creeping flow conditions and it becomes unstable due to inertia, however small in magnitude in comparison to the viscous forces. The long wave instability caused can be suppressed by geometric arrangement in which the more viscous fluid occupies bigger region in channel cross-section than the less viscous fluid. And the shortwave instability is known to be suppressed by introduction of an interfacial tension. However, the numerical analysis of Yiantsios and Higgins [4], which examined the disturbances of any arbitrary wavelength, found that the most dangerous disturbance has wavelength of the order of the channel width, such that the most critical wavenumber, made non-dimensionalized by channel width, $k \sim O(1)$.

The polymeric fluid introduces elasticity behavior through a finite relaxation time, which generates normal stresses in shear flow. For two fluids with equal densities and viscosities, the flow can become unstable if the elasticities are unmatched. For an interface perturbed from flat state, the normal stress applies an axial (in the direction of flow) force on the interface depending on the magnitude of fluid elasticity. For two fluids with dissimilar elasticities, there exists a net force on the interface, creating a flow disturbance. In the long wave limit, the flow becomes unstable when the thickness of more elastic fluid is smaller than that for the less elastic fluid [6,7]. In other extreme of shortwave disturbances, the asymptotic analysis carried out by Renardy [8] finds that the flow is also unstable to shortwave mode originating from a jump in the first normal stress difference across the interface. Interestingly, for the viscoelastic fluids, both the longwave and shortwave instabilities occur in the inertia-less flow, in the limit of zero Reynolds number. Thus, mere elasticity stratification is sufficient to destabilize the interface between two fluids in plane Couette flow and plane Poiseuille flow in the creeping flow limit [6,8–10]. While the longwave instability can be suppressed by confining less elastic fluid to a thinner region than the more elastic fluid, the shortwave instability may be stabilized by the presence of an interfacial tension. However, it is known that the most critical disturbance for which the growth rate is maximum is the one with wavelength comparable to the channel width, *i.e.* $k \sim O(1)$, where k is the axial wavenumber made non-dimensionalized by channel width [9–12]. The experimental studies to investigate interfacial instability in channel flow of two superposed viscoelastic fluids have also been carried out to verify the existence of purely elastic instability [13–16]. By imposing temporal disturbances of controlled wavelengths, the experiments confirm the numerical predictions that the most dangerous disturbance is of wavelength comparable to channel width, *i.e.* maximum growth rate is for wavenumber $k \sim O(1)$.

For a two-layer plane Couette flow of upper convected Maxwell (UCM) fluids, Renardy [8] constructed stability maps in $W_1 - W_2$ plane indicating regions of stable and unstable flow, where W_1 and W_2 are Weissenberg numbers of two fluids. However, the author considered only the shortwave disturbances into account. The stability diagram shows that while the unmatched elasticity leads to shortwave instability, the case of very dissimilar elasticities, *i.e.* $W_1 \gg W_2$ or $W_1 \ll W_2$ is found to be stable. Further, due to the shortwave nature of instability, the surface tension tends to suppress the interfacial instability. Interestingly, the viscoelastic fluids introduce a new length scale, the fluid relaxation time multiplied by the velocity scale. Hence, the classification of longwave (wavelength greater than channel width) and shortwave (wavelength smaller than channel width) modes needs to be redefined. In order to understand the role of the length scale originating from the fluid relaxation time, Miller and Rallison [17] analyzed the interfacial instability of two superposed UCM fluids in the limit of very high fluid elasticity, such that $W \gg k \gg 1$. Here, even though the disturbance profile does not decay away from the interface, the growth rate is independent of the fluid thickness, meaning the instability is localized at the interface. However, unlike shortwave mode, this interfacial instability is not suppressed by the surface tension, even when it is infinite. The underlying physical mechanism of the interfacial instability in highly elastic fluids is not properly understood.

For pressure driven flows in co-extrusion, the interface stability in two symmetrically superposed viscoelastic fluids under plane Poiseuille flow has been carried out by Wilson and Rallison [18]. The shortwave instability due to non-zero jump in normal stress across the interface is observed for unmatched fluid elasticities. The numerical analysis finds the maximum growth rate for sinuous perturbations at moderate wavenumbers, $k \sim O(1)$. The plane Poiseuille flow has also been analyzed for interfacial instability when multiple layers of fluid are superposed [19–21]. These analyses address the role of simultaneous viscosity and elasticity stratification coupled with the effect of thickness ratio of fluid layers and various arrangements for stable interface have been proposed. In earlier analyses of plane Poiseuille flow, the attention is restricted to fixed values of elasticity stratification and analyses have been carried out for limited values of flow Weissenberg numbers focusing mainly on varying viscosity and layer thickness ratios. Moreover, the Couette–Poiseuille flow with adverse pressure gradient has not been studied. In the present study, we analyze the plate-driven flow superposed with pressure gradient to address the interfacial instability excited within the coating cup in the fiber coating process. The two-layer fluids are considered as the UCM fluids, suitable to describe the polymer melt and highly concentrated polymeric solutions. The numerical analysis examines the disturbance with any arbitrary wavelength and constructs stability diagram in $W_1 - W_2$ plane covering a range of fluids with varying extent of elasticity stratification.

2. Problem formulation

The system consists of two superposed viscoelastic fluids with viscosities η_1 and η_2, relaxation times λ_1 and λ_2, confined between two flat plates separated by distance L. While the bottom plate at $y = 0$ is held stationary, the top plate, at $y = L$, moves with a constant velocity V in the positive x-direction. The parameter δ indicates the fraction of channel width occupied by the bottom fluid (referred to as fluid 2); thus $(1 - \delta)$ is the fraction of channel width occupied by the top fluid (fluid 1). The densities of both fluids are assumed to be the same. The co-ordinate system and the schematic of the flow geometry is shown in Fig. 1. We consider the plane Couette flow with zero pressure gradient [Fig. 1(a)] as well as the Couette–Poiseuille flow with an adverse pressure gradient [Fig. 1(b)] and a favorable pressure gradient [Fig. 1(c)]. The emphasis of the study is on the role of non-zero pressure gradient in the interfacial instability. In the present analysis, the quantities are made dimensionless as follows: distance by L, velocities by V, time by L/V, pressure and viscoelastic stresses by $\eta_2 V/L$. The flow is assumed to be creeping flow owing to high viscosities of the viscoelastic fluids.

The dimensionless continuity and momentum conservation equations for the inertia-less flow are:

$$\nabla \cdot \mathbf{v}^{(\alpha)} = 0, \tag{1}$$

$$-\nabla p^{(\alpha)} + \nabla \cdot \boldsymbol{\tau}^{(\alpha)} = 0. \tag{2}$$

Here, α denotes the index for the fluid: $\alpha = 1$ for the top fluid and $\alpha = 2$ for the bottom fluid. The dynamics of the viscoelastic fluid is described with the help of upper convected Maxwell (UCM) model:

$$\begin{aligned} &W_\alpha[\partial_t \boldsymbol{\tau}^{(\alpha)} + \mathbf{v}^{(\alpha)} \cdot \nabla \boldsymbol{\tau}^{(\alpha)} - \boldsymbol{\tau}^{(\alpha)} \cdot \nabla \mathbf{v}^{a} - (\nabla \mathbf{v}^{(\alpha)})^T \cdot \boldsymbol{\tau}^{(\alpha)}] + \boldsymbol{\tau}^{(\alpha)} \\ &\quad = m_\alpha[\nabla \mathbf{v}^{(\alpha)} + (\nabla \mathbf{v}^{(\alpha)})^T], \end{aligned} \tag{3}$$

where superscript T indicates the transpose and $m_\alpha = \eta_\alpha/\eta_2$, the viscosity of fluid 'α' made non-dimensionalized by bottom fluid viscosity η_2. The Weissenberg number for fluid 'α', an indicative of its elasticity, is defined as $W_\alpha = \lambda_\alpha V/L$, where λ is the relaxation time of the polymeric fluid.