Journal of Non-Newtonian Fluid Mechanics 224 (2015) 40–50

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

A scaling rule for the flow mobility of a power-law fluid through unidirectional fibrous porous media

Hai Long Liu [a], Moon Kwang Um [b], Wook Ryol Hwang [c,*]

[a] School of Energy and Power Engineering, Jiangsu University, 301 Xuefu Road 301, Zhenjiang, 212-013, China
[b] Composite Materials Research Center, Korea Institute of Materials Science (KIMS), Changwon, 642-831, Republic of Korea
[c] School of Mechanical Engineering, Research Center for Aircraft Parts Technology (ReCAPT), Gyeongsang National University, Jinju-daero 501, Jinju, 660-701, Republic of Korea

ARTICLE INFO

Article history:
Received 20 October 2014
Revised 3 August 2015
Accepted 4 August 2015
Available online 12 August 2015

Keywords:
Flows in porous media
Permeability (Mobility)
Power-law fluid
Scaling rule
Lubrication approximation

ABSTRACT

We propose a scaling rule for the flow mobility prediction of a power-law fluid in unidirectional fibrous porous media by separating rheological contribution from geometrical factor. Starting from the analogy to a simple circular pipe flow, we find mobility dependence on pressure drop with the help of lubrication theory for flows of a power-law fluid through arrays of square and hexagonal packing cylinders. There appears a nearly unique intersection point independent of rheological parameters in the mobility and pressure drop space, once scaled with the consistency index, and the intersection point coordinate is expressed by rheological and geometrical factors separately. Further simplification is achieved by approximating the rheological contribution by a constant that scales linearly with the consistency index. The proposed mobility estimation scheme facilitates a simple but reasonably accurate estimation and it has been verified by extensive numerical simulations for various pressure drops, rheological parameters, porosities and fibrous porous architectures.

1. Introduction

Flows of rheologically complex fluids in porous media are of great interest in a large variety of industries such as chemically enhanced oil recovery, reactions in fluidized and fixed beds, coating of textile, paper and fibrous mats, filtration in various fields, and more interestingly in thermoplastic composites manufacturing with fibrous porous fibers as the reinforcement [1–6]. In such processes, rheologically complex fluids are often involved with non-Newtonian behaviors in various degrees, including shear-dependent viscosity or fluid elasticity. One example is multifunctional composites manufacturing with particle/epoxy or CNT/epoxy suspension in the liquid composite molding process to improve thermal/electrical conductivity or mechanical performance [7]. Not only the particle suspensions, but also CNT/epoxy suspension, once well dispersed, also shows shear-thinning behavior over a typical shear rate range of the process. Elasticity may be negligible even for CNT/epoxy suspension in this process due to low shear rate in the process and relatively high viscosity of the epoxy [8]. The permeability or mobility with a non-Newtonian fluid is influenced by coupled effects of both fluid rheology and porous architecture; hence a priori prediction of flow rate through the porous media by a given pressure drop, which is particularly desirable in designing and optimizing the processes, cannot be made explicitly in this case [9, 10]. In the present work, we specifically focus on the power-law fluid only among others to take the shear-thinning behavior into account. In this case, the shear-dependent viscosity can be written as follows:

$$\eta(\dot{\gamma}) = m\dot{\gamma}^{n-1}, \tag{1}$$

where the shear rate $\dot{\gamma} = (2\boldsymbol{D}:\boldsymbol{D})^{1/2}$ is the second invariant of the rate-of-deformation tensor $2\boldsymbol{D}$ in a flow, the symbol m is the consistency index and n denotes the power-law index.

For a Newtonian fluid, the Darcy's equation is commonly employed to model slow flows thorough the porous media:

$$\bar{u} = -\frac{K}{\eta}\frac{\Delta p}{L}, \tag{2}$$

where $\bar{u}$, η, K, Δp and L are the superficial velocity, the viscosity, the permeability, the pressure drop and the dimension of a sample in the flow direction. The permeability is a constant that measures the fluid conductance in porous media and a characteristic property of the porous medium with a Newtonian fluid. The one-dimensional form in Eq. (2) can be naturally extended to multiple dimensions by introducing the second-order permeability tensor. However, the permeability with a non-Newtonian (shear-thinning in this work) fluid is no longer a constant and does not follow the tensor transformation rule due to non-linearity in the shear-rate dependent viscosity. There is an interplay between complex porous geometry and fluid rheology.

* Corresponding author. Tel.: +82 55 772 1628; fax: +82 55 772 1566.
E-mail address: wrhwang@gnu.ac.kr (W.R. Hwang).

http://dx.doi.org/10.1016/j.jnnfm.2015.08.002
0377-0257/

To circumvent this complexity in the permeability, Bruscheke and Advani [11] introduced a concept of the mobility M, a ratio of the permeability to the viscosity, as a dumped parameter combining effects of porous geometry and fluid rheology, and the Darcy's law is then written as

$$\bar{u} = -M\frac{\Delta p}{L}. \tag{3}$$

The mobility measures the amount of flow rate for a given pressure difference. They also introduced a normalized mobility M' for a power-law fluid as a dimensionless form of the mobility

$$M' = \frac{m}{Q^{1-n}R^{2n}}M. \tag{4}$$

The quantity Q denotes the flow rate and R is the fiber radius in fibrous porous media. As the power-law index n approaches to unity, the normalized mobility reduces to the normalized permeability K/R^2 of a Newtonian fluid. The normalized mobility M' can be considered as a reasonable choice for the dimensionless mobility measure, since it does not depend on the pressure drop Δp. (This independence can be easily shown from Eq. (4) and the fact that M scales with $\Delta p^{(1-n)/n}$ and Q scales with $\Delta p^{1/n}$.) However, the normalized mobility M' has little physical interpretation, since it does not scale with the flow rate: i.e., the higher M' does not mean the higher Q. Moreover, it behaves significantly complicated such that a priori estimation of M' is not straightforward [12].

Flow in porous media of a Newtonian fluid has been investigated extensively by experimental measurements [13–16] or theoretical predictions [17,18], and it has been relatively well understood. Especially, some efforts have been made to develop concise models to predict the permeability of various arrangements of fibrous structures, which have been verified to be successful in case of a Newtonian fluid [19–21]. In contrast much less is known about non-Newtonian fluid flows through porous media [2,3]. Bruschke and Advani [11] developed a close form solution analytically for the power-law fluid flow through square and hexagonal arrangements for a wide range of porosity and their analytical approximation has been verified by experiments of Sadiq et al. [22]. Lundström et al. [23] derived an expression of the apparent permeability for the power-law fluid flow perpendicular to unidirectional fibers and comparison of their model with numerical simulations has been presented as well. Similar researches have been performed by Spelt et al. [24,25] and Idris et al. [26], which have been extended to anisotropic porous structures consisting of regular aligned elliptical cylinders [27–29]. Especially, Woods et al. [27] presented correlation between apparent permeability for a power-law fluid to the corresponding permeability for a Newtonian fluid by introducing a simple scaling with the power-law coefficient, the length scale and the velocity scale, though porous architecture and fluid rheology have not been clearly separated in their model. We remark that the power-law model, though not perfect for excessive viscosity in very low shear regimes, has been successfully applied to model the flow resistance or mobility of shear-thinning fluid flows in porous media as was verified in literatures [3,4,22].

The objective of the present study is to propose a much simpler but reasonably accurate method in predicting the mobility and its dependence of the pressure drop of a power-law fluid in fibrous porous media by separating the contribution of fluid rheology from that of the porous architecture along with further approximation such that the rheological factor is replaced by a constant with properly scaled variables. From the analogy to a simple circular pipe flow, a closed form expression of the mobility in terms of the pressure drop has been derived using the lubrication theory for regular fiber packing problems. Recognizing the presence of a nearly unique intersection point in the scaled mobility and the pressure drop space, we propose a mobility estimation method based on the intersection point location, which can be expressed explicitly in terms of rheological and geometrical factors. The proposed mobility estimation has been verified by extensive numerical simulations. The paper is organized as follows: in Section 2.1, we describe the scaling concept for the mobility using a simple pressure-driven pipe flow, which is composed of (i) the intersection point in the scaled mobility-pressure drop space, (ii) separation of rheological and geometrical factors in expressions of the intersection point coordinate, and (iii) approximation of rheological factors by a constant. In Section 2.2, we derive analytical solutions for the mobility for a power-law fluid flow in unidirectional fibrous media and then we apply the scaling concept to find the closed form expression of geometrical and rheological factors along with the approximation. In Section 3, we verify the proposed scaling rule by comparison with extensive numerical simulations for square, hexagonal and random packing fiber arrays.

2. Theoretical modeling

2.1. A scaling rule in a simple pressure-driven circular pipe flow

2.1.1. Observations on the mobility behavior

The idea of a scaling rule in this work for a power-law fluid in porous media can be best introduced by a simple example of a pressure-driven flow in a circular pipe. In this problem, the fully developed flow rate Q is expressed in terms of the radius R of the circular pipe and the pressure gradient dp/dx in the flow direction x as follows:

$$Q = \frac{\pi R^3}{(1/n+3)}\left(\frac{dp}{dx}\frac{R}{2m}\right)^{\frac{1}{n}}. \tag{5}$$

By taking the velocity $\bar{u}$ as the mean velocity over the cross section, the flow mobility M can be obtained by an analogy with the Darcy's law of flows in porous media (Eq. 3):

$$M = \frac{R}{(1/n+3)}\left(\frac{R}{2m}\right)^{\frac{1}{n}}\left(\frac{dp}{dx}\right)^{\frac{1-n}{n}}. \tag{6}$$

The mobility of the power-law fluid converges to the Newtonian limit ($M_{Newt.} = R^2/8m$) as n approaches to the unity and it does not depend on the pressure gradient any more in this case. For the power-law index n less than the unity, the mobility M scales linearly with the pressure gradient dp/dx in the log–log plot with the slope of $(1-n)/n$. Fig. 1(a) presents the mobility M in Eq. (6) as a function of the pressure gradient with two different pipe radii $R = 0.1$ and 1 for three different power-law indices $n = 0.3$, 0.5 and 1 for two different consistency indices $m = 1$ and 5. In the log–log mobility-pressure drop space, the slope is completely determined by the power-law index and, for a given geometry R and a consistency index m, there appear a unique intersection point for various power-law indices.

In Fig. 1(b), the mobility coordinate has been scaled with the consistency index m and the pressure gradient coordinate is scaled with $1/m$; then the mobility dependence on the consistency index has disappeared in the scaled coordinate. This means that mobility curves for various rheological parameters (m and n) for a given geometry intersect at a single point in the *scaled* $M - dp/dx$ space in Fig. 1(b) and the intersection point depends only on the geometrical variable R. The corresponding pressure gradient to this intersection point may be considered as a critical pressure gradient, since the mobility increases with the amount of shear-thinning above the critical point and thereby enhances the flow rate under a given pressure gradient, and vice versa. We remark that all these observations are quite similar to flow behaviors of a power-law fluid in 2D fiber beds and 3D packed spheres reported by the authors' previous works [12,30]. The mobility curves for a given geometry (radius) intersecting at a unique point in the scaled $M - dp/dx$ space implies that, once the intersection point is known, the mobility dependence on the pressure drop can be reproduced easily for any values of the pressure gradient, since the slope of the mobility is completely determined by the power-law indices $(dp/dx)^{(1-n)/n}$.