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Isogeometric static analysis of laminated composite plane beams by using refined zigzag theory

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ABSTRACT

An attempt has been made here for the isogeometric static analysis of the laminated composite plane beams by using refined zigzag theory. In this study; instead of the standart finite elements, which use polynomial shape functions, an isogeometric refined zigzag finite element (IGRZF) has been developed, that gives the opportunity to get the exact beam geometries directly from a computer aided design (CAD) software, Rhinoceros. To provide less computational effort, the refined zigzag theory has been introduced, that make IGRZF independent from the number of layers considered. The aforementioned finite element has been implemented in an in-house Mathematica code, which can handle both thin and thick beams without the problem of shear locking and does not require shear correction factors. Using this approach, various sandwich beams have been analyzed and the obtained results are compared with other reliable published results for various aspect ratios and support types.

1. Introduction

Composite materials, that are typically set up with piled up layers of different materials, offer superior strength performance and weight properties from the other materials, thus the area of their use has increased in aircraft and aerospace vehicles as well as automative, naval and civil structures. Composite laminates often exhibit significant transverse shear deformation than isotropic counterparts due to their low transverse shear stiffness characteristic. Transverse shear deformation between strong fiber-rich and weak resin-rich layers through the thickness causes delamination to occur in multiple locations such as core/face sheet interface, laminae interface and core [1]. It is essential to predict accurate stress and strain fields to determine where the delamination begins. One method to achieve this is the Finite Element Analysis (FEA). Researchers have developed finite elements according to two approaches, which are (1) the Equivalent Single Layer approach (ESL) and (2) the Layerwise approach (LW).

The ESL theories are those in which a heterogeneous laminated beam is treated as a statically equivalent, single layer having a complex constitutive behavior, reducing the 3-D continuum problem to a 2-D problem [2]. The main drawback of this method is that the continuity of transverse shear stresses and normal stresses along the thickness direction, is not always assured. The simplest ESL laminate theory is the classical laminated theory (CLT) which is an extension of the Euler-Bernoulli beam theory to laminated composite beams. CLT is applicable to slender beams only, since it neglects both transverse shear and transverse normal effects and deformation is due entirely to bending and in-plane stretching. The first order shear deformation beam theory (FOBT) is the extension of Timoshenko beam theory for laminated beams [3]. FOBT assumes that the transverse shear strain to be constant with respect to the thickness coordinate; which causes the zero shear stress conditions on the top and bottom surfaces of the beam to be violated. Moreover; this assumption results in predicting the transverse shear stresses to exhibit erroneous discontinuities along the layer interfaces of the laminated beam due solely to the two plies that have different shear modulus. To account for the discrepancy between the actual stress and the assumed constant stress state; shear correction factors are required which are difficult to determine for arbitrarily laminated composite beams, since they depend not only on the lamination and geometric parameters, but also on the loading and boundary conditions. Even with a neatly chosen shear correction factor, the FOBT under predicts axial and transverse shear stress. To avoid the need for shear correction coefficients used in the FOBT; higher order shear deformation theories are developed by expanding the in-plane displacement up to the cubic term in the thickness coordinate to have quadratic variation of the transverse shear strains and transverse shear stresses through each layer. The third order laminated beam theory of Reddy [4] accounts for traction free conditions at the top and bottom surfaces of the beam using constitutive relations. Ferreira et al. [5] proposed a meshless method based on the multiquadric radial basis functions by using the third order theory of Reddy [4] for laminated composites. Several studies also have been performed on higher order shear deformation theories [6–9]. Arya et al. [10], Sayyad and Ghugal [11], Ghugal and Shikhare [12], Vo and Thai [13], Nazargah et al. [14]

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developed displacement based higher order shear deformation theories considering trigonometric functions in terms of thickness coordinate to account the effects of transverse shear and normal deformations for the bending analysis of laminated composites and sandwich beams. Even with higher order ESL theories, the results are often inaccurate when the layer properties vary drastically through the thickness, due to the increased presence of transverse shear effects. The reason for this discrepancy in results is due to the C^1 continuity of in-plane displacements through the thickness assumed in the model, leading to double valued interlaminar shear stresses at the lamina interfaces.

In contrast to ESL models, the LW models assume the displacement field C^0 continuous through the laminate thickness, thereby allowing for continuous interlaminar stresses at the layer interfaces, so that a correct representation of the cross-sectional warping associated with deformation is attained. The laminate thickness is subdivided into computational layers, each viewed as an equivalent, single, homogenous layer, then the displacement and contact conditions at interfaces are imposed as constraint conditions. To obtain the desired degree of in-plane displacement variation through the thickness is obtained by adding more subdivisions which increases the number of dependent unknowns.

Computational costs of the LW models that increase with the number of computational layers lead the researchers to develop alternative models. Partial layerwise models, the so-called zig-zag models, are developed to account for the discrete-layer effects of transverse shear. Here, the zig-zag pattern of the axial displacements given by the exact elasticity solutions is provided by introducing a zig-zag displacement function for each layer. Importantly, the number of kinematic variables in zig-zag theories is independent of the number of layers. Carrera in [15] states that Lekhnitskii [16] was the first to propose a zig-zag theory that described the C_z^0 requirements by solving a boundary value problem related to the compatibility equations which were written in terms of a stress function and the evaluation of transverse stresses does not require any post-processing procedure, such as the use of Hooke's law or integration of 3D equilibrium equations. Apart from the method by Lekhnitskii [16], two other independent and different zig-zag theories were given by Ambartsumian [17] and Reissner [18]. Ambartsumian theory (AMT) is the one that has mostly influenced the development of multilayered beam, plate and shell theories since it uses a well-known theory such as the Reissner-Mindlin. Whitney [19] first applied and extended AMT to anisotropic and symmetrical and nonsymmetrical plates. A second work on the application of AMT was given by Rath and Das [20] by extending the work done by Whitney [19] to doubly curved shells. A particular case of AMT was dealt by Di Sciuva and co-authors [21]. Di Sciuva [22] also introduced further enhancements to the zigzag model by adding a cubic in-plane displacement to the zigzag function. However, the theory of Di Sciuva require C^1 continuous shape functions for formulating suitable finite elements which makes the theory less attractive than the C^0 continuous displacement interpolations associated with Timoshenko type theories. Moreover, the theory suffers from an anomaly at a clamped support condition, where the cross-sectional area integral of the transverse shear stress, obtained from constitutive relations, does not correspond to the total shear force. Averill [23] modified the Di Sciuva approach by starting with Timoshenko theory, introducing a penalty term to enforce the continuity of transverse shear stress across the cross-section. Although Averill's model [23] results in C^0 continuous kinematics which makes it suitable for efficient finite element interpolations, the model similarly breaks down at the clamped end. Tessler et al. [24] developed a Refined Zigzag Theory (RZT) that incorporates with Di Sciuva [22] and Averill's [23] theories and devoid of the anomalies encountered. In contrast to Di Sciuva [22], the theory of Tessler et al. [24] does not enforce the shear-stress continuity and the zigzag function is set to vanish only on the outer surfaces of the laminate which makes the function contribute to the local distortion of the beam's cross-section in each layer. Gherlone et al. [25] and Onate et al. [26] have developed standart C^0 finite elements based on the RZT for multilayered composite and sandwich laminates. The lack of enforcement of the interface shear stress continuity in RZT leads to discontinuities of the transverse shear stress along the layer interfaces and it needs the results to be much improved by computing transverse shear stresses 'a posteriori' from the axial stress field using the Cauchy's equilibrium equation. However, the post-processed transverse shear stresses do not satisfy the underlying equilibrium equations of the theory anymore and this makes the technique to be variationally inconsistent. The necessity of the aforementioned post-processing technique in RZT has been overcome by Tessler [27], employing a mixed-field formulation for laminated composite and sandwich beams, labeled RZT$^{(m)}$, that is based on Reissner's Mixed Variational Theorem [18]. The formulation of RZT$^{(m)}$ was further extended to 2-D plates [28]. Groh and Tessler [29] have recently developed computationally efficient nine degree-of freedom (dof) and eight-dof shear locking-free beam elements using RZT$^{(m)}$ to obtain accurate predictions of stresses within interply resin-rich zones which is crucial for predicting the onset and propagation of delaminations in laminated composites.

The laminated composite structures have been studied with different analytical techniques and numerical approaches such as the traditional finite element methods (TFEM). However, the TFEM uses Lagrange or Hermitian type polynomial shape functions to discretize the geometry which will yield a geometrical error, especially in complex geometries how much the mesh is refined. Another disadvantage of TFEM is that, it is incompatible with the computer aided design (CAD) softwares, i.e., Rhinoceros, AutoCAD, Maya that use Non-uniform Rational B-splines (NURBS) as a basis to represent the geometry. To close the gap between the two technologies and integrate the NURBS based CAD tools into the finite element analysis, isogeometric analysis (IGA) is recently introduced by Hughes et al. [30]. The isoparametric concept in TFEM, in which the discretization of geometry is executed by the basis chosen to approximate the unknown solution fields, has been reversed back in IGA such that the selected basis capable of exactly representing the geometry, will also be used as a basis for the fields to be approximated [31]. Moreover, IGA takes the advantage by using NURBS basis functions which provide higher continuity of derivatives than polynomial basis used in TFEMs. Thanks to these features, isogeometric analysis has been used for solving different mechanical problems on laminated plates and shells [32–36]. However, few works [14,37] are available in the literature that uses the IGA for the static analysis of laminated composite beams based on the advanced and refined theories.

In the present study, a new isogeometric finite element (IGRZF), based on RZT, is introduced. The employed RZT was first introduced by Tessler et al. [24]. The IGRZF has properties that make it to be computationally low cost and effective such as the independency of the number of layers considered, free from shear locking and geometrical error. Nevertheless, the lack of enforcement of the interface shear-stress continuity in RZT, leads to discontinuities of the transverse shear stress along the layer interfaces, however, the results are much improved by computing transverse shear stresses "a posteriori" from the axial stress field using the Cauchy's equilibrium equation.

The rest of this paper is organized as follows. Section 2 introduces the theoretical basis behind the RZT, computation of the zigzag function and the derivation of the generalized constitutive equations. In Section 3, a brief introduction to B-Splines and NURBS is given and the approximation of the geometry and of the field variables within the isogeometric analysis framework is presented. The accuracy and the effectiveness of the proposed IGRZF are verified through several numerical examples in Section 4. The numerical results exhibit good agreements with the available published results and the elasticity solutions.