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Non-Newtonian slender drops in a simple shear flow

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ABSTRACT

The deformation and breakup of a power-law non-Newtonian slender drop in a Newtonian liquid in a simple shear and creeping flow has been theoretically studied. The problem is governed by three dimensionless parameters: The capillary number (Ca), the viscosity ratio (λ), and the power-law index (n) of the non-Newtonian drop. The results show: (a) slender drops exist if $n \le 1$ only, these drops have pointed ends; (b) for the same strength of the flow, Newtonian drops are more elongated than shear thinning drops; and (c) for the same viscosity ratio, shear thinning drops are more difficult to break than Newtonian drops.

1. Introduction

The deformation of a non-buoyant Newtonian drop in a Newtonian liquid under creeping flow conditions is governed by two dimensionless parameters. The capillary number Ca, the ratio of the external viscous force (that tends to deform the drop) to the surface tension force (that tends to keep the drop spherical), and λ, the ratio of the viscosity of the drop to that of the external fluid. Small deformations are obtained at $Ca \ll 1$. In this paper, however, we shall deal with slender bodies only, which can be realized when $Ca \gg 1$ and $\lambda \ll 1$. A summary of studies in this area can be found in reviews by Rallison [1], Stone [2] and Briscoe et al. [3].

A slender body theory for a drop in creeping flow was first suggested by Taylor [4], who studied the deformation of a drop in an axisymmetric extensional flow, where the cross-section of the drop is circular. The theory, which was later refined by Buckmaster [5,6], Acrivos & Lo [7] and others, shows that the local radius of the slender drop has a parabolic shape with pointed ends. According to the theory, the critical capillary number needed for breakup increases as the viscosity ratio decreases. For an inviscid drop, where $\lambda = 0$, a stable steady shape is always possible, and breakup was not predicted.

Hinch & Acrivos [8] extended the slender body theory for a two-dimensional extensional flow, a case where the drop cross-section is not circular. Their results showed that the drop cross-section can be approximated by an ellipse with an axis ratio of 1.5. The deformation of the drop and the critical strength of the flow, for drop breakup, were essentially identical to those obtained for the axisymmetric extensional flow case, where the cross-section of the drop is circular. Based on these results, together with the fact that experiments have shown that slender drops in simple shear flow do not align with the principal axes of strain (at 45° to the direction of the flow) but rather with the direction of the flow, Hinch & Acrivos [9] developed a model for drop deformation in simple shear flow with the assumption that the drop has a circular local cross-section.

The theoretical solution of Hinch & Acrivos [9] suggests an S-shaped drop with pointed ends. As the capillary number increases, the drop becomes thinner and longer and its inclination with the direction of the flow decreases. Also, the critical capillary number needed for breakup increases as the viscosity ratio decreases. In contrast to extensional flow, where no steady shape (stable or unstable) can be obtained beyond a critical flow strength; in simple shear flow, a steady shape (stable or unstable) exits for all strengths of the flow. The theoretical model of Hinch & Acrivos [9], for the deformation of inviscid drops, was experimentally confirmed by Canedo et al. [10], Rust and Manga [11] and Müller-Fischer et al. [12,13]. As expected the cross-section of the drop was not circular, but elliptical with an axis ratio of 1.4 [10]. Furthermore, the theoretical prediction of Hinch & Acrivos [9] for drop breakup (due to fracture, as opposed to tip streaming), agrees well with previous experiments conducted by Grace [14]. Drop breakup by tip streaming, in simple shear flow, has been observed by many investigators. Today, it is well established that surfactants moving

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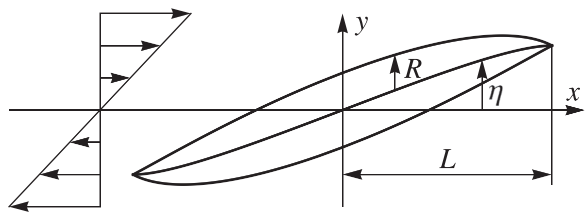

Fig. 1. A slender drop in a simple shear flow: $R(x,t)$ is the local radius, $\eta(x,t)$ is the centerline position and $L(t)$ is the half-length of the drop.

towards the tips of the drop and accumulating there are responsible for this phenomenon (De Bruijn, [15]; Janssen et al., [16]; Renardy et al. [17]). The theoretical analysis of Hinch & Acrivos [9] was extended by Janssen et al. [18] for a drop highly confined between two parallel walls.

So far, our discussion has been limited to slender drops in Newtonian systems. This is far from reality in many industrial applications, such as the mixing of polymer blends, where non-Newtonian effects such as shear thinning and elasticity are present. There are a large number of theoretical and experimental studies on drop deformation and breakup in simple shear flow in non-Newtonian systems. Unfortunately, these studies are restricted to small to medium deformations; see for example the publication of Aggrawal & Sarkar [19] which contains a large literature review. Despite the fact that there is no experimental study primarily dedicated to slender drops in simple shear flow in non-Newtonian systems, the recent report of Boufarguine et al. [20] suggest that shear thinning non-Newtonian drops embedded in Newtonian liquids deform less than Newtonian drops.

The only theoretical studies dealing with slender drops in non-Newtonian systems can be found in our previous reports (Favelukis & Nir, [21]; Favelukis et al., [22]; [23]), but they are restricted to extensional flow only. Thus, it is the purpose of this paper to extend the slender-body theoretical analysis of Hinch & Acrivos [9] for simple shear flow, to non-Newtonian systems. As a first step, we shall include here non-Newtonian effects inside (but not outside) the slender drop and we show that, similar to extensional flow, a local analysis close to the tip of the drop predicts that only Newtonian and shear thinning drops can exist. The rheology of the drop is described by the simple power-law model, which offers mathematical simplicity. Since the flow within most of the volume of the slender drop is dominated by shear components, the power-law model can provide a good description of shear thinning or thickening effects.

2. The governing equations

2.1. The flow outside the drop

Assume a slender drop embedded in an infinite viscous Newtonian fluid (see Fig. 1) subjected to a simple shear and creeping flow, which at infinity is of the form:

$$v_x = Ey,\, v_y = 0,\, v_z = 0 \tag{1}$$

where $E > 0$ is a constant shear rate and let the origin coincide with the drop center. Following Hinch & Acrivos [9], we also assume that the drop has a local circular cross-section in the (y, z) plane of radius $R(x,t)$, a displacement of the center-line position above the direction of the flow $\eta(x,t)$, and a half-length $L(t)$. To a first approximation, the disturbed velocity profile, outside and near the surface of the drop, obtained in terms of singularities distributed along the centerline of the drop, is parallel to the undisturbed flow. Adding the flow disturbance, found in Hinch & Acrivos [9], to Eq. (1), we find:

$$v_x \approx E\left(\eta + r\sin\theta + \frac{R^2}{r}\sin\theta\right),\, v_y \approx 0,\, v_z \approx 0 \tag{2}$$

Here r and θ are local polar coordinates with respect to the centerline of the drop, such that $y = \eta + r\sin\theta$ and $z = r\cos\theta$. At the next order the disturbance lies entirely in the yz plane and expressions for the disturbance velocities together with the pressure and stress profiles can be found in Hinch & Acrivos [9].

2.2. The flow inside the drop

The incompressible non-Newtonian fluid inside the drop obeys the simple power-law model:

$$\tau = m\left|\sqrt{\frac{1}{2}(\dot{\gamma} : \dot{\gamma})}\right|^{n-1}\dot{\gamma} \tag{3}$$

where τ is the viscous stress tensor, $\dot{\gamma}$ is the rate of deformation tensor and m and n are model parameters. For a Newtonian fluid $n = 1$ and m becomes the Newtonian viscosity. For a shear thinning (pseudo-plastic) fluid $n < 1$ while for a shear thickening (dilatant) fluid $n > 1$.

The flow within the drop is characterized by a pressure gradient and with a velocity at the boundary given by Eq. (2) at $r = R$. In the absence of an explicit solution for the velocity profile that is uniformly valid along the entire drop (Bird et al., [24]), an approximation of the inner profile is given by a superposition of the two modes, drag and pressure driven flows:

$$v_x = E(\eta + 2r\sin\theta) - \left(\frac{n}{1+n}\right)\left(\frac{R}{2m}\frac{\partial p}{\partial x}\right)^{1/n} R\left[1 - \left(\frac{r}{R}\right)^{1+1/n}\right] \tag{4}$$

where p is the pressure inside the drop, assumed uniform at each cross section. Note that this expression reduces to the Newtonian case ($n = 1$) given by Hinch & Acrivos [9].

For the similar case of a non-Newtonian slender drop in an extensional flow (Favelukis et al. [22]), the superposition assumption equals the exact solution. In a two-dimensional flow, the superposition approximation and the exact solution are the same when the upper and lower surfaces move in the same direction with equal velocity (as it is near the drop tip) or move in opposite directions with equal velocity (as it is near center of the drop). It is anticipated that, in the intermediate region, the deviation will not be appreciable due to the relatively small internal pressure gradients in shear flow.

The volumetric flow rate through each axial cross section along the drop, obtained by integrating the velocity profile, is:

$$Q = \pi R^2\left[E\eta - \left(\frac{n}{1+3n}\right)\left(\frac{R}{2m}\frac{\partial p}{\partial x}\right)^{1/n} R\right] \tag{5}$$

This flow is balanced by the rate of volume change, measured from the center of the drop, at $x = 0$, to that cross section position:

$$Q = -\pi\frac{\partial}{\partial t}\int_0^x R^2 \mathrm{d}x \tag{6}$$

Equating the last two equations yields the pressure profile inside the drop:

$$p = p_0(t) + 2wmE^n\int_0^x \frac{1}{R}\left[\frac{\eta}{R} + \frac{1}{R^3}\frac{\partial}{\partial(Et)}\int_0^x R^2 \mathrm{d}x\right]^n \mathrm{d}x,$$

$$w = \left(\frac{1+3n}{n}\right)^n \tag{7}$$

where $p_0(t)$ is the unknown pressure at the center of the drop. Note that, at the steady state, the volumetric flow rate vanishes everywhere. For an inviscid drop, $m = 0$, the pressure inside the drop is uniform, and at steady-state it is also constant.