Journal of Non-Newtonian Fluid Mechanics 228 (2016) 31–37

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

Short Communication

Determination of draw resonance onsets in tension-controlled viscoelastic spinning process using transient frequency response method

Ilyoung Kwon [a], Myung-Suk Chun [b], Hyun Wook Jung [a,*], Jae Chun Hyun [a]

[a] Department of Chemical and Biological Engineering, Korea University, Seoul 136-713, Republic of Korea
[b] Complex Fluids Laboratory, National Agenda Research Division, Korea Institute of Science and Technology (KIST), Seoul 136-791, Republic of Korea

ARTICLE INFO

Article history:
Received 18 July 2015
Revised 27 November 2015
Accepted 17 December 2015
Available online 22 December 2015

Keywords:
Viscoelastic spinning
Draw resonance instability
Constant-tension condition
Frequency response
Critical onsets

ABSTRACT

Draw resonance onsets of actual viscoelastic spinning processes with secondary forces operated by constant draw ratio are determined via transient frequency response simulations under a constant tension or force boundary condition, which causes the system to be always stable. Transfer function data between the spinline velocity and the tension at the take-up position in the frequency domain, which are transformed from the transient responses of the take-up velocity with respect to a step change of the take-up force, play a key role in the determination of the onset points for various spinning cases. It is confirmed that the critical draw ratios established in this study are almost the same as those decided by linear stability or direct transient simulation methods for velocity-controlled spinning systems. The transfer function data for the stability analysis can be readily and beneficially applied to an analysis of sensitivity of a spinning process with respect to a sinusoidal disturbance.

1. Introduction

Melt spinning process is one of representative extensional deformation polymer processes. The polymer melt that is extruded from the spinneret nozzle is uniaxially elongated and then taken up at the take-up position after being solidified inside the spinline region, under the specified draw ratio or drawing force and cooling boundary conditions. Owing to the interesting nature of the flow-instability-related dynamic characteristics of this process, it has thus far attracted considerable attention from both theoretical and experimental viewpoints in academia and industries [1–3].

The draw resonance instability, which is a supercritical Hopf bifurcation [4,5], occurring in this process is characterized mainly by periodic oscillations of spinline variables such as the spinline tension, fiber diameter, and fluid velocity beyond the critical draw ratio. The draw ratio (D_R) is defined as the dimensionless ratio of the velocity at the take-up position to that at the spinneret positions. Draw resonance instability inevitably occurs in an actual spinning process operated under the constant draw ratio condition, and it is triggered by the variation of tension inside the spinline. However, if a constant force or constant tension condition is manipulated instead of the constant draw ratio operation, the spinning system would be completely insensitive to any disturbances, thereby resulting in it being always stable (Fig. 1) [1,6–8].

Since the successful pioneering of fundamental modeling of the spinning processes by Kase group [9,10], and Pearson group [11], various ingenious methods for determining marginal stability of draw resonance have been explored on the basis of linear and nonlinear stability theories for Newtonian and viscoelastic fluids. It is well-known that the case of simple Newtonian spinning under isothermal and no-secondary force conditions has the critical draw ratio of 20.21 corresponding to a draw resonance onset. Critical onset points that demarcate stable and draw resonance states are of course significantly affected by the cooling process, viscoelasticity including dichotomous extensional-thickening and -thinning behaviors, and secondary force conditions [12–14].

Several algorithms are available for determining draw resonance onsets. For example, the linear stability method can be used to decide the neutral state from an eigenfunction system assembled by introducing infinitesimal perturbations to nonlinear governing equations [8,13–21]. A stability criterion employing kinematic waves that penetrate the entire spinline region [22–25] and a frequency response method of a linearized system in combination with closed-loop transfer functions [26] are both suitable candidates for theoretically predicting onsets. Further, a direct transient simulation of nonlinear governing equations under the constant draw ratio condition [13,18,27] can be applied to find

* Corresponding author. Tel.: +82 2 3290 3306.
E-mail address: hwjung@grtrkr.korea.ac.kr (H.W. Jung).

http://dx.doi.org/10.1016/j.jnnfm.2015.12.006
0377-0257/

draw resonance onsets, which interestingly exhibits nonlinear limit cycle behaviors in unstable states after tedious time-consuming calculations.

In this study, draw resonance onsets for viscoelastic spinning processes with secondary forces are newly determined from transient simulation data obtained from nonlinear governing equations during a relatively short time period, i.e., nearly one residence time scale, under the constant force condition (i.e., not the conventional constant draw ratio), guaranteeing always stable states. A closed-loop transfer function concept of the frequency response developed by Kase and Araki [28] for the case of simple viscous spinning is adopted to obtain Nyquist plots comprising real and imaginary parts of the transfer function between the spinline velocity as an output and the step-changed tension as an input disturbance at the take-up position under different viscoelasticity and secondary force conditions. It is also verified that the transfer function data employed in deciding the onsets can effectively contribute to the sensitivity analysis of the spinning process.

2. Governing equations for viscoelastic spinning flows

Isothermal spinning processes with Phan-Thien and Tanner (PTT) viscoelastic fluids [29] were numerically analyzed on the basis of the following dimensionless governing equations.

Continuity equation:

$$\frac{\partial a}{\partial t} + \frac{\partial}{\partial x}(av) = 0, \tag{1a}$$

$$a \equiv \frac{A}{A_0}, v \equiv \frac{V}{V_0}, x \equiv \frac{z}{L}, t \equiv \frac{T}{L/V_0}, \tag{1b}$$

where A is the spinline area, V is the spinline velocity, z is the flow direction coordinate, T is the time, and L is the spinline distance from the spinneret to the take-up. The subscript 0 indicates the spinneret position.

Equation of motion:

$$Re\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \frac{1}{a}\frac{\partial}{\partial x}(a(\tau_{xx} - \tau_{rr})) + G, \tag{2a}$$

$$\tau_{xx} \equiv \frac{\sigma_{zz}L}{\eta V_0}, \tau_{rr} \equiv \frac{\sigma_{rr}L}{\eta V_0}, \ Re \equiv \frac{\rho V_0 L}{\eta}, G \equiv \frac{\rho g L^2}{\eta V_0}, \tag{2b}$$

where Re is the Reynolds number, G is the dimensionless gravity coefficient, σ_{zz} is the axial stress, σ_{rr} is the radial stress, η is the fluid viscosity, ρ is the fluid density, and g is the gravitational accelerator. It is noted that secondary forces such as inertia and gravity forces included in this equation make the tension along the spinline different.

Constitutive equation (PTT fluids):

$$K\tau_{xx} + De\left(\frac{\partial \tau_{xx}}{\partial t} + v\frac{\partial \tau_{xx}}{\partial x} - 2(1-\xi)\tau_{xx}\frac{\partial v}{\partial x}\right) = 2\frac{\partial v}{\partial x}, \tag{3a}$$

$$K\tau_{rr} + De\left(\frac{\partial \tau_{rr}}{\partial t} + v\frac{\partial \tau_{rr}}{\partial x} + (1-\xi)\tau_{rr}\frac{\partial v}{\partial x}\right) = -\frac{\partial v}{\partial x}, \tag{3b}$$

$$K \equiv \exp\left(\varepsilon De\left(\tau_{xx} + \tau_{rr}\right)\right), De \equiv \frac{\lambda V_0}{L}, \tag{3c}$$

where, ε and ξ are parameters for PTT fluids, λ is the material relaxation time, and De is the Deborah number.

The constant flow rate at the spinneret and constant force at the take-up are applied as boundary conditions to completely solve the above equations along the spinline.

Boundary conditions and initial condition:

$$a_0(t) = v_0(t) = 1 \text{ at } x = 0, \tag{4}$$

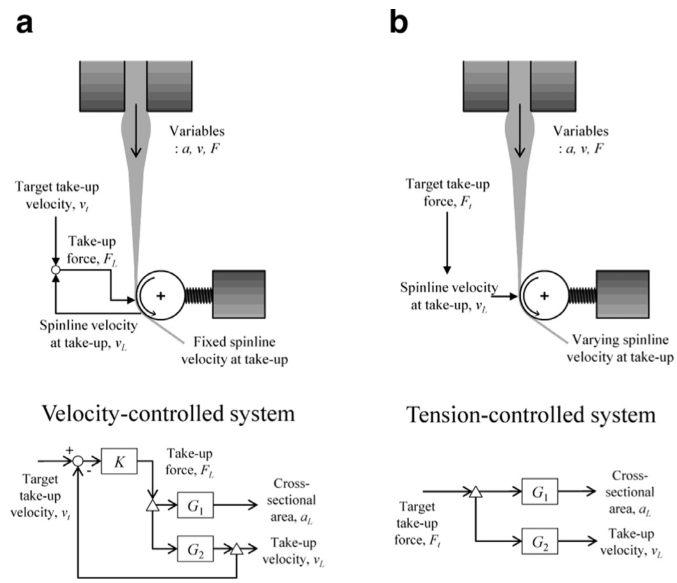

Fig. 1. Schematic diagrams and conceptual control loops of spinning process operated under (a) constant velocity and (b) constant tension conditions at take-up. G_1 and G_2 are transfer functions and K is the proportional gain.

$$F_L(t) = \begin{cases} F_L^0 = a_L(\tau_{xx,L} - \tau_{rr,L}) & \text{when } t = 0 \\ F_L^0(1+\delta) & \text{when } t > 0 \end{cases} \text{ at } x = 1, \tag{5}$$

where F is the spinline tension; δ is a small disturbance; and the subscript L and superscript 0 indicate the take-up position and initial steady-state value, respectively. After a step change is imposed on the tension at the take-up (Eq. (5)), transient responses of state variables are obtained until their profiles arrive at new steady states. As mentioned above, the application of this take-up boundary condition in the transient simulation directly makes it possible to readily and accurately examine the stability and sensitivity of the spinning system in the frequency domain by preventing further mathematical manipulations.

Spatial derivatives in the governing equations were discretized by the second-order central finite difference method. It should be noted that the backward difference method was applied for convective inertia terms in order to stabilize the solutions numerically [12]. Further, the implicit Euler method was implemented for the discretization of time derivatives. The total number of spatial nodal points was chosen as 4,000 to ensure the accuracy of the numerical values.

3. Frequency response via transient simulation to determine draw resonance onsets

From the frequency response using linearized spinning formations for generalized Newtonian fluids as explored by Kase and Araki [28], it was found that the transfer function between the take-up velocity and the spinline force had a key link in the occurrence of draw resonance. In previous cases [26,28], nonlinear governing equations for spinning flows were required to be additionally linearized in order to find onsets by the linear stability or transfer function method via the frequency response. In the present study, the above-mentioned transfer function data were directly obtained from nonlinear transient simulations of the spinning process accompanied by the viscoelasticity and secondary forces, without the further linearization step. Unlike the case of transient simulation under the constant draw ratio condition, no significant numerical problem will occur, because the constant force condition results in the spinning system being always stable.

Fig. 1 shows a schematic comparison between two spinning modes induced separately by the constant draw ratio and constant