Journal of Non-Newtonian Fluid Mechanics 220 (2015) 3–21

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: http://www.elsevier.com/locate/jnnfm

Effect of aiding-buoyancy on mixed-convection from a heated cylinder in Bingham plastic fluids

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ARTICLE INFO

Article history:
Received 19 March 2014
Received in revised form 29 May 2014
Accepted 4 June 2014
Available online 23 June 2014

Keywords:
Mixed convection
Bingham fluid
Cylinder
Richardson number
Prandtl number

ABSTRACT

Mixed convection heat transfer from a heated horizontal circular cylinder in Bingham plastic fluids has been studied numerically over wide ranges of the governing parameters as follows: Reynolds number, $0.1 \leqslant Re \leqslant 40$; Prandtl number, $1 \leqslant Pr \leqslant 100$; Richardson number, $0 \leqslant Ri \leqslant 2$ and Bingham number, $0 \leqslant Bn \leqslant 10$. Extensive results on the flow and heat transfer characteristics are presented in terms of the streamlines and isotherm contours in the close proximity of the cylinder and the distribution of pressure and the Nusselt number over the surface of the cylinder. The gross behavior is described in terms of the drag coefficient and average Nusselt number as functions of the above-noted influencing parameters. In addition, the morphology of the flow domain in terms of the size and shape of (and their dependence on the governing parameters) the yielded- and unyielded regions separated by the so-called yield surfaces is also analyzed. The momentum and thermal boundary layers progressively thin with the increasing values of each of *Re*, *Pr*, *Bn* and *Ri*. Thus, it stands to reason that the rate of heat transfer should bear a positive dependence on each of these parameters. The results reported herein elucidate this fundamental dependence. Finally, the heat transfer results are consolidated by choosing a slightly modified velocity and viscosity scales thereby enabling a satisfactory correlation between the modified Nusselt number and Reynolds number.

1. Introduction

It is readily acknowledged that many multiphase and/or structured fluids like foams, emulsions and suspensions encountered in a range of engineering applications display visco-plastic flow behavior [1–3]. Typical examples include processed foods and chocolates [4], toiletries and cosmetics [5], drilling muds and lubricating greases [6], building materials [7], etc. Other examples can be found in Refs. [1–3,8,9]. A visco-plastic substance is characterized by its dual nature, *i.e.*, when the externally applied stress is below its yield stress, it deforms like an elastic solid. Once the magnitude of the applied stress exceeds the value of the fluid yield stress, it deforms like a fluid with constant (Bingham plastic) or shear-thinning (Herschel–Bulkley) viscosity [8,9]. Naturally, such a dual nature makes convective transport in these fluids rather difficult, for only molecular transport occurs in the solid-like unyielded regions which may form a substantial part of the flow domain. This, in turn, can limit the overall rate of convective transport in such fluids. Thus, for instance, not only their mixing in batch systems tends to be rather difficult [9,10], heat transfer also is severely impeded in these fluids during their heating or cooling in numerous process engineering applications. In spite of their frequent occurrence in a range of settings, very little is known about their heat transfer characteristics. A cursory inspection of the available body of information clearly reveals that the bulk of the literature pertains to duct flows [11,12], porous media flows [8,13]. Indeed, very little information is available on heat transfer in visco-plastic fluids in the so-called external boundary layer-type of flows such as that over a sphere and a circular cylinder. Depending upon the prevailing flow conditions, heat transfer may occur in the forced- or free- or the mixed-convection regimes. In the mixed-convection regime, the relative importance of the forced- and free-convection contributions is expressed using the familiar Richardson number, *Ri*, which is defined as $Ri = Gr/Re^2$. Thus, the two limiting cases of $Ri \to \infty$ and $Ri \to 0$ correspond to the pure free- and forced-convection flow regimes respectively. On the other hand, the values of the Richardson number of order unity correspond to the mixed-convection regime where the externally imposed velocity is comparable to that due to the buoyancy effects. Therefore, under such conditions, both free- and forced convection contributions must be considered in a given application. Additional complications arise in the case of mixed convection depending upon the direction of the forced flow with reference to the direction of gravity. Thus, both flows may be in the same

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http://dx.doi.org/10.1016/j.jnnfm.2014.06.006
0377-0257/

Nomenclature

Bn	Bingham number (dimensionless)
Bn^*	modified Bingham number based on effective velocity (dimensionless)
C	specific heat of fluid (J/kg K)
C_D	total drag coefficient (dimensionless)
C_{DP}	pressure drag coefficient (dimensionless)
C_p	pressure coefficient $(= 2(p_s - p_\infty)/\rho U_\infty^2)$ (dimensionless)
d	diameter of the cylinder (m)
D_∞	diameter of fictitious computational domain (dimensionless)
Gr	Grashof number (dimensionless)
g	acceleration due to gravity (m/s^2)
h	local heat transfer coefficient (W/m^2 K)
j_h	Colburn j_h-factor (dimensionless)
k	thermal conductivity of fluid (W/m K)
m	growth rate parameter (dimensionless)
N_p	number of elements on the surface of cylinder (dimensionless)
Nu_L	local Nusselt number (dimensionless)
Nu	average Nusselt number (dimensionless)
Nu_f	average Nusselt number in the forced convection regime (dimensionless)
p	pressure (Pa)
p_s	pressure at a point on the surface of cylinder (Pa)
p_∞	free stream pressure (Pa)
Pr	Prandtl number (dimensionless)
Pr^*	modified Prandtl number (dimensionless)
Re	Reynolds number (dimensionless)
Re^*	modified Reynolds number based on effective velocity (dimensionless)
Re^{**}	modified Reynolds number based on effective velocity and viscosity (dimensionless)
Ri	Richardson number (dimensionless)
T	dimensionless temperature $(= \frac{T'-T_\infty}{T_w-T_\infty})$
T_∞	fluid temperature in the free stream (K)
T_w	temperature on the surface of the cylinder (K)
U_∞	far away free stream velocity (m/s)
U_{eff}	effective velocity (m/s)
U_x	x-component of velocity (dimensionless)
U_y	y-component of velocity (dimensionless)

Greek symbols

β	coefficient of volumetric expansion (1/K)
$\dot{\gamma}$	rate of strain tensor (dimensionless)
η	representative viscosity of fluid (Pa s)
η_{eff}	effective viscosity (Pa s)
θ	angular displacement from the front stagnation point ($\theta = 0$), degree
μ_B	Bingham plastic viscosity of fluid (Pa s)
μ_{yield}	yielding viscosity of fluid (Pa s)
ρ	density of the fluid (kg/m^3)
ρ_∞	density of fluid at the reference temperature T_∞ (kg/m^3)
τ	extra stress tensor (Pa)
τ_0	fluid yield stress (Pa)

Subscripts

f	forced convection
w	surface of cylinder
∞	free stream fluid

Superscript

$'$	dimensional variable

direction (aiding buoyancy), or may be in the opposite direction (opposing buoyancy) or the two may be oriented normal to each other (cross-buoyancy). Hence, unlike in the case of free or forced convection, the resulting flow patterns and hence heat transfer characteristics can vary significantly in aiding-, opposing- and cross-buoyancy mixed convection in Bingham plastic fluids from a heated horizontal cylinder. Furthermore, such model configurations not only merit a systematic study in their own right, but these also represent an idealization of numerous industrial applications like flow in tubular and pin-type heat exchangers, measuring probes and continuous thermal treatment of food particles. Also, this work complements our recent work on the forced- and free-convection heat transfer from a circular cylinder [14,15]. Prior to the discussion of the new results obtained in this work, it is desirable to recount the key results available for mixed-convection heat transfer from a horizontal cylinder in Newtonian and power-law fluids. This discussion, in turn, serves as a reference case to draw inferences about the role of yield stress on heat transfer from a cylinder in the mixed convection regime.

Over the years, mixed convection from a heated cylinder has been studied extensively in Newtonian and, somewhat less widely, in power-law fluids over wide ranges of conditions. Since most of these studies have been reviewed elsewhere [16–18], only the main points are recapitulated here. Significant literature is now available on mixed-convection heat transfer from a circular cylinder in Newtonian fluids like air, water and viscous oils over wide ranges of Rayleigh number spanning both the laminar and turbulent flow conditions as well as in the different regimes of mixed convection [16–19]. Consequently, based on a combination of the approximate analytical (boundary layer approximation), numerical and experimental studies, it is now possible to estimate the value of the average Nusselt number for a circular cylinder in the mixed-convection regime in a new application. For instance, based on their experimental results, Hatton et al. [20] put forward the following expression for the aiding-buoyancy configuration for mixed-convection from a horizontal isothermal cylinder.

$$Nu = 0.384 + 0.581 Re_{eff}^{0.439} \tag{1}$$

where the effective Reynolds number, Re_{eff}, is based on the vectorial sum of the external and buoyancy-induced velocities. In turn, the effective Reynolds number, Re_{eff}, is related to that based on the forced convection velocity, Re, as follows:

$$Re_{eff} = Re[1 + 2.06\chi\cos\theta + 1.06\chi^2] \tag{2}$$

where $\chi = \frac{Ra^{0.48}}{Re}$.

Evidently, in the limit of the forced convection, the Grashof number (hence the Rayleigh number, Ra) is identically zero and the two definitions of the Reynolds number coincide, *i.e.*, $Re_{eff} = Re$ *i.e.*, $\chi = 0$. In Eq. (2), θ is the angle between the directions of the imposed velocity and the gravity vector. Thus, for aiding-buoyancy case, $\theta = 0$ and $\theta = 180$ denotes the case of the opposing-buoyancy, and $\theta = \pi/2$ corresponds to the cross-buoyancy configuration. Finally, Eq. (2) is based on the experimental data for air ($Pr = 0.71$) for an isothermal cylinder spanning the following ranges of conditions: $10^{-2} \leqslant Re \leqslant 45$ and $10^{-3} \leqslant Ra \leqslant 10$. On the other hand, Jackson and Yen [21] were able to correlate their mixed-convection experimental results for a cylinder in air as follows:

$$\frac{Nu}{Nu_f} = (1 + Ri)^{0.2133} \tag{3}$$