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# An admissible function for vibration and flutter studies of FG cylindrical shells with arbitrary edge conditions using characteristic orthogonal polynomials

Huagang Lin[a], Dengqing Cao[a,*], Chonghui Shao[b]

[a] School of Astronautics, Harbin Institute of Technology, Harbin 150001, China
[b] Harbin Turbine Company Limited, Harbin 150046, China

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ABSTRACT

A general approach for the vibration and aeroelastic stability of the functionally graded cylindrical shell with arbitrary boundary conditions is firstly presented. The Sanders' shell theory, a steady-state heat transfer equation and the piston theory are employed to establish the motion equation, where the thermo-mechanical properties of material are set to be location- and temperature- dependent. The orthogonal polynomials series generated by employing the Gram–Schmidt process are taken as the admissible functions to express the general formulations of displacement. Moreover, the artificial spring technique is introduced to simulate the elastic constraints imposed on the cylinders' edges. The frequency equations are derived considering the strain energy of artificial springs during the Rayleigh–Ritz procedure, and the motion equation of cylindrical shells subjected to combined thermal and aerodynamic loads is established based on the Hamilton principle. A few comparisons for the frequency and critical flutter pressure are performed to validate the proposed approach. The influences of the volume fraction, thermal gradient, boundary conditions and spring stiffness on the flutter characteristics are highlighted. This paper overcomes the limitations of previous vibration and flutter studies which are confined to the structure under simply supported or clamped boundaries.

## 1. Introduction

The aeroelastic stability of common components in the axial flow is very critical during the design of skin shells and panels for the high-performance vehicles, aerospace aircraft and high speed missiles [1]. Functionally graded materials (FGM) are a kind of composite materials intentionally designed to possess some desirable properties for specific applications, especially for the structures under the high-temperature environment [2]. The outstanding features of employing the materials are that these composite materials can survive the high speed aerodynamic environment and thermal gradient loads, while maintaining their strength and integrity. The cylindrical shell structures are widely used by the missiles and rockets in the supersonic airflow. Generally, the coupling model of structure and aerodynamic loads is analyzed in a range of altitude and velocity which defines the aircraft flight envelope [3]. However, it is difficult in predicting the vibration and flutter behavior of these cylindrical shells with general elastic support conditions such as riveting or the looseness of the support constraint. The key of the problem lies in accurately extracting the frequency and mode characteristic of shells with the general support edges, serving for the subsequent flutter analysis.

A number of studies have been conducted to investigate the vibration characteristics of panels and shells. Loy et al. [4] studied the free vibration of FGM structure, and the effects of the volume fractions on the frequency were discussed. Sofiyev [5–7] presented an analytical approach to investigate the vibration problems of FG cylindrical shells and truncated conical shells. Fuchiyama and Noda [8] used a finite element method to discuss the transient heat conduction of FGM structure, and found that when the volumetric ratio was linearly distributed, the maximum stress would have the minimum value. Bagherizaseh et al. [9] investigated the buckling of shells made of FGM with the Pasternak elastic foundation under a temperature field. The aero-thermoelastic stability of panels was studied by Navazi and Haddadpour [10] based on the Galerkin method. The static and dynamic stabilities of panels under the thermal loads and aerodynamic pressure were discussed by Sohn and Kim [11] and Prakash and Ganapathi [12]. The flutter problem of carbon nanotube reinforced composite structures were investigated by Kiani [13] and Asadi and Wang [14], respectively. Chen and Li [15] investigated the aeroelastic properties and nonlinear response of a cylindrical shell subjected to an

* Corresponding author.
*E-mail address:* dqcao@hit.edu.cn (D. Cao).

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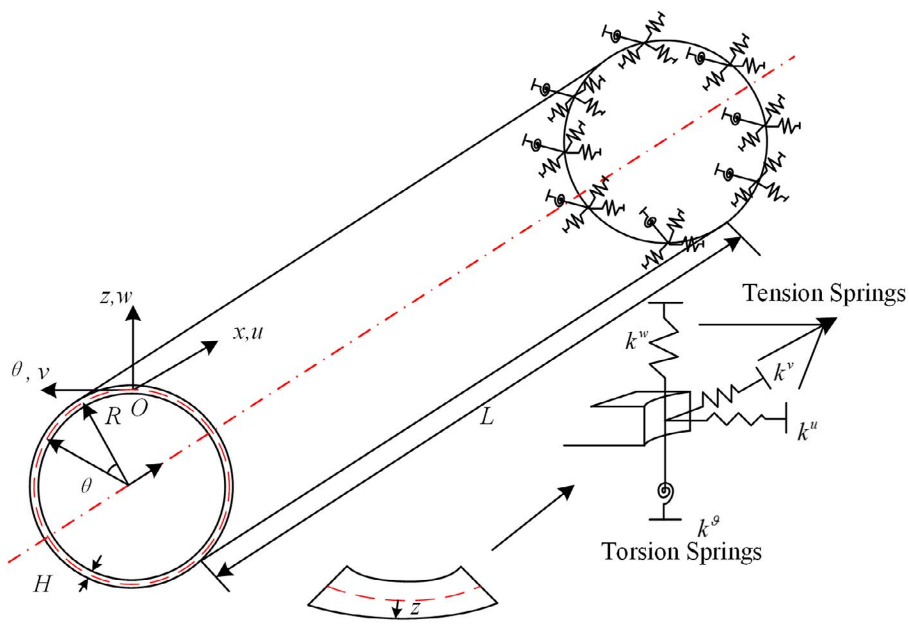


**Fig. 1.** FG cylindrical shell with arbitrary support boundary.

aerodynamic pressure. Haddadpour et al. [16] investigated the influences of temperature environment and internal load on the flutter characteristics of the shell with simply supported boundary. Surveying the open literatures, few studies on the dynamic stability of FG cylindrical shell are found. Torki et al. [17] discussed the dynamic stability of FG cylindrical shells using the first-order shear deformation theory. Bochkarev et al. [18] investigated the flutter problem of FG cylindrical shells subjected to the external supersonic flow, where the effects of the functionally graded materials, thermal load and airflow velocity on the aeroelastic stability were discussed. Sabri and Lakis [19] presented a hybrid FEM for flutter prediction of FG cylindrical shell. Dai et al. [20] presented a review of FG cylindrical structures under coupled physical interactions, including fluid-solid coupling and stability analysis. We find, these studies are focused on the dynamic behaviors of structure with simple supported boundary, but most structures in practical engineering are supported or connected with a complex support or arbitrary boundary conditions. It is necessary to achieve new approach and solution to solve the vibration and aeroelastic stability of FG panel or shell under arbitrary boundary.

There are also few researches on the simulation for the support boundary. The vibration problem of the shell with elastic supports were investigated by Loveday and Rogers [21], Wang et al. [22]. Amabili and Garziera [23] used the artificial spring to study the vibration of shell with complex boundary constraints, in which the mode of the less-restrained condition was applied to expand the solutions of displacement. The free vibration of composite laminated shell under general elastic boundary was studied by Li [24], where the displacements were expressed by a Fourier cosine series, and the effects of boundary support stiffness on frequency were illustrated. Zhang et al. [25] investigated the vibration problem of a plate with elastic restrained edges and internal supports. Sun et al. [26] and Liu et al. [27] presented a general approach to study the vibration characteristics of rotating shell with elastic boundary, where Rayleigh–Ritz process was used to derive the frequency equation. The studies on the simulation of elastic support are limited to the free vibration of isotropic structure, and there are few literatures dealing with some complicated problems, such as the analysis of buckling, flutter and response of FG composites shell. Moreover, the admissible functions play an important role in solving the vibration problem of composite structures with arbitrary boundaries. This study is motivated by the need to meet actual engineering conditions for the vibration and flutter problem of FG cylindrical shell with arbitrary boundary.

The novelty of this study is to present a general theoretical approach for the stability analysis of the FG cylindrical shell with arbitrary boundary conditions. The material property of FG cylindrical shell is assumed to be temperature-dependant, and graded through the thickness according to the power-law distribution. A novel solution of computing the vibration characteristics of the cylindrical shell with arbitrary boundary is proposed, where the admissible functions are constructed by a set of characteristic orthogonal polynomials generated directly by employing the Gram–Schmidt process, and the edges support is modeled by the artificial springs. The simulation of arbitrary boundary can be realized by setting the stiffness value of artificial springs to be extremely large, small or arbitrary. To simulate a temperature rise, two cases of uniform and temperature gradient elevations through the thickness are considered. The dynamic stability boundaries are defined by linear flutter theory with eigenvalue analysis. A few comparisons for the frequency and critical freestream flutter pressure with the results obtained from the FEM and technical literatures are performed to validate the proposed approach. Some numerical results are given to illustrate the influence of the variations of spring stiffness on the frequency and flutter behavior. The solution broadens the research field of cylindrical shells with arbitrary supports, e.g. the study of stability, flutter and response, which are confined to the frequency and mode in the past.

## 2. Theoretical formulation

Consider the FG cylindrical shell in a supersonic airflow, the length, radius and thickness are $L$, $R$ and $H$, respectively and the airflow direction is along the $x$ axis, shown in Fig. 1. The displacement of a point in the middle surface of cylindrical shell are expressed as $u$, $v$ and $w$ in the coordinate system $O\text{–}x\theta z$. The artificial springs including three direction's tension spring and one direction's torsion spring, are distributed on both edges of the cylindrical shell. The cylindrical shells having arbitrary boundary can be carried out by adjusting the stiffness of the artificial springs.

### 2.1. Functionally graded cylindrical shell

The artificial springs are introduced to describe the connect between the cylindrical shell and the supporting members. The stiffness of artificial springs including the axial, circumferential, radial and rotation directions are $k^u$,$k^v$,$k^w$ and $k^\vartheta$, respectively in Fig. 2. Setting