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Squeeze plane flow of viscoplastic Bingham material

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Abstract

We develop an asymptotic solution for the plane squeeze flow of a viscoplastic medium. The standard lubrication-style expansions of the problem predict plug speed which varies slowly in the principal flow direction. This variation implies that the plug region cannot be truly unyielded. Our solution shows that this region is a pseudo-plug region in which the leading order equation predicts a plug, but really it is weakly yielded at higher order. We follow the asymptotic technique suggested earlier by Balmforth & Craster [25] and Frigaard & Ryan [26]. We also provide comparison between analytical solution and numerical computations using the augmented Lagrangian method. The results of computations show presence of unyielded regions near the two stagnation points of flow close to centers of plates (that was found earlier by many researchers) and new additional unyielded regions at the outer edge of the material (both for short and long plates). The obtained analytical expression for the squeeze force is in a good agreement with the numerical results and previous results of Mitsoulis & Matsoukas (2003).

Keywords: viscoplastic fluid, Bingham medium, lubrication theory, squeeze flow, pseudo-yield surface, pseudo-plug.

1 Introduction.

Viscoplastic materials behave as rigid solids, when the imposed stress is smaller than the yield stress, and flow as fluids otherwise. The flow field is thus divided into unyielded (rigid) and yielded (fluid) zones. Two types of rigid zones are traditionally distinguished: the stagnation (dead) zones, where the medium is at rest, and the plug regions, where the medium moves as a rigid body. The surface separating a rigid from a fluid zone is known as a yield surface. The location and shape of the latter must be determined as part of the solution of the flow problem.

In this paper we develop an asymptotic solution for the squeeze flow of Bingham material. Squeeze flows are flows in which a material is deformed between two parallel plates approaching each other. Analysis of viscoplastic fluid flows in geometries with small aspect ratio have a long history. Most of the works in the literature deal with the so-called «lubrication paradox» for yield stress fluids, which refers to the existence or not-existence of a true unyielded plug region. Lipscomb & Denn probably first have shown [1] that the usual lubrication approximations in squeeze flow of Bingham fluids leads to a paradox. They argued that true rigid plug regions should not exist in complex geometries, with a reference to a squeeze flow. The argument is that adoption of classical lubrication, scaling techniques leads to the prediction that the unyielded plug region moves with a speed which slowly varies in the principal flow direction. This variation implies that the plug region cannot be truly unyielded. Such regions have been termed pseudo-plug regions and the boundaries are called either pseudo-yield surfaces or fake yield surfaces. The squeeze flow problem is well-studied [2]-[24], detailed review can be found in [18]. The excellent work [14] provided the locations of rigid zones either qualitatively or quantitatively. They showed that unyielded material must exist only around the two stagnation points of flow at the center of the disks and their numerical results confirmed this.

The asymptotic procedure suggested by Balmforth & Craster [25] and Frigaard & Ryan [26] leads to success in resolution of "lubrication paradox" and allows to construct the consistent solution for thin-layer problems. Here we shall apply this technique to obtain a consistent thin-layer solution for the squeeze problem of viscoplastic material.

Further we compare our analytical solution with the numerical one. The main difficulty in the numerical simulation of viscoplastic fluid flow is related to the non-differentiable form of constitutive law and inability to evaluate the stresses in regions where the material has not yielded. There are two principal approaches that have been proposed in the literature to overcome the mathematical problem of viscoplastic fluid flow. The first one, known as regularization method, consists in approximating the constitutive equation by a smoother one.