Journal of Non-Newtonian Fluid Mechanics 220 (2015) 162–174

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: http://www.elsevier.com/locate/jnnfm

Plane flow of thixotropic elasto-viscoplastic materials through a 1:4 sudden expansion

Fernanda B. Link [a], Sérgio Frey [a,1], Roney L. Thompson [b], Mônica F. Naccache [c], Paulo R. de Souza Mendes [c,*]

[a] Department of Mechanical Engineering, Federal University of Rio Grande do Sul, Rua Sarmento Leite 425, Porto Alegre, RS 90050-170, Brazil
[b] LMTA-PGMEC, Department of Mechanical Engineering, Universidade Federal Fluminense, Rua Passo da Pátria 156, Niterói, RJ 24210-240, Brazil
[c] Department of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente 225, Rio de Janeiro, RJ 22453-900, Brazil

ARTICLE INFO

Article history:
Received 28 May 2014
Received in revised form 3 February 2015
Accepted 18 February 2015
Available online 19 March 2015

Keywords:
Thixotropy
Elasto-viscoplasticity
Oldroyd-B model
Sudden expansion
Galerkin least-squares method

ABSTRACT

A numerical investigation of an elasto-viscoplastic thixotropic fluid flowing through a 1:4 plane expansion is performed, using a recently proposed constitutive equation. The conservation equations are solved using a four-field Galerkin least-squares formulation in terms of the extra stress, pressure, velocity, and structure parameter—a scalar quantity that represents the structuring level of the material microstructure. The focus is on determining the effect of thixotropy, elasticity and viscoplasticity on the topology of yielded and unyielded regions of the expansion, on the field of structuring level, and on the field of elastic strain. Relevant ranges of the relaxation time, yield stress, and thixotropy characteristic time are investigated. The numerical results reveal significant effects of these parameters. The trends observed are physically sound and in accordance with the related literature.

1. Introduction

Thixotropic materials can be found both in nature and in industrial applications, and are present in a large number of our daily activities. Emulsions, gels, paints, drilling fluids, food products, and mineral slurries are some examples of possibly thixotropic materials. Typically these materials consist of dispersions possessing a microstructure that governs their macroscopic behavior in response to applied stresses. These structured materials exhibit a complex non-Newtonian behavior, usually including viscoelasticity, yield stress and thixotropy. Thixotropic materials present a time delay between a change in the applied stress and the response of the microstructure (breakdown or buildup). Many thixotropic constitutive models have been proposed over the last decades, but testing and validation information is rather scarce.

Mujumdar et al. [1] developed a model to describe the rheological behavior of thixotropic fluids with yield stress and elasticity, based on the kinetic process responsible for structure changes in the fluid. A structure parameter λ that indicates the level of structure of the material was defined. The rheological response is a function of the material structure, and time effects are taken into account in the evolution equation for the structure parameter. In the constitutive equation proposed, the total stress is an arithmetic mean of an elastic and a viscous term, weighted by the structure parameter λ. When the material structure breaks down, a viscous behavior occurs, and elasticity decreases. The results obtained were in fair agreement with frequency sweep experiments.

The thixotropic behavior of fluids was thoroughly reviewed by Barnes [2] and Mewis and Wagner [3,4]. In these articles, typical experiments and fluid responses are presented and analyzed, and the relation between thixotropy and viscoelasticity, reversibility and modeling are also discussed. It is observed that elasticity can be present in thixotropic fluids, especially in the gel phase. Mewis and Wagner [4] observed that different approaches have been used for modeling thixotropy.

In this paper we analyze the performance of the constitutive equation for elasto-viscoplastic thixotropic fluids proposed by de Souza Mendes [5] in the flow through a plane expansion. This geometry has been studied numerically and experimentally by several authors and using some of the well known models for purely viscous or viscoelastic fluids, e.g. Papanastasiou, Power-law and Oldroyd-B models (e.g. [6–12]). In these studies, the effect of different parameters (such as the Reynolds number, Yield

* Corresponding author.
E-mail addresses: feulink@mecanica.ufrgs.br (F.B. Link), frey@mecanica.ufrgs.br (S. Frey), rthompson@id.uff.br (R.L. Thompson), naccache@puc-rio.br (M.F. Nac cache), pmendes@puc-rio.br (P.R. de Souza Mendes).
[1] Visiting Professor in Department of Chemical and Biomolecular Engineering, Rice 6100 Main St., Houston, TX 77005, United States.

http://dx.doi.org/10.1016/j.jnnfm.2015.02.009
0377-0257/

number, and Deborah number) on the flow pattern and on the head loss.

The constitutive equation employed in the present work is based on the upper-convected Oldroyd-B constitutive equation, modified to include structuring level dependence in the shear modulus and in the relaxation and retardation viscosities. This model is able to describe the typical characteristics of structured fluids, such as stress overshoots in constant shear rate experiments, non-monotonic flow curves, bifurcation of the viscosity function, performs correctly in small and large amplitude oscillatory flows, and is thermodynamically consistent [13,14]. The models proposed by de Souza Mendes [15], de Souza Mendes [5] differ significantly from the ones previously available in the literature and present a number of improvements, as thoroughly discussed in de Souza Mendes and Thompson [16].

The structure parameter is governed by an evolution equation that has no diffusion term, and hence it requires special care in its numerical approximation. The numerical modeling of the governing equations is based on a four-field Galerkin least-squares formulation in terms of the structure parameter, viscoelastic stress, pressure, and velocity [17].

The formulation mentioned above is used to perform a numerical investigation of the steady flow of an incompressible structured fluid flowing through a 1:4 plane expansion. Inertia is neglected, and thixotropic and elastic effects are evaluated for a relevant range of the governing parameters.

2. The analysis

2.1. The flow domain

The flow domain is illustrated in Fig. 1. It is composed of two parallel-plate channels assembled sequentially to form a planar sudden expansion. The spacing between plates and axial length of the upstream channel are $2H_u$ and L_u, respectively, and the corresponding dimensions of the downstream channel are $2H_d$ and L_d (Fig. 1). In the research reported here we focused our attention on the study of the influence of the dynamical and rheological parameters, and so we kept the geometry fixed. The following values of the geometrical parameters were employed for all cases investigated:

$$\frac{H_d}{H_u} = 4; \quad \frac{L_u}{H_u} = \frac{L_d}{H_u} = 50 \tag{1}$$

These axial lengths ensure that fully-developed flow conditions are attained before the end of both channels, for all cases investigated.

2.2. Constitutive model

The mechanical behavior of the flowing material is described by the constitutive model given in de Souza Mendes [5]. Its stress equation is essentially the same as the one of the Oldroyd-B model, except that the model parameters are allowed to vary with the *structure parameter* λ which is a measure of the structuring level of the microstructure, such that $\lambda = 0$ when the material is fully unstructured and $\lambda = 1$ when it is fully structured. For convenience in the numerical scheme, here we employ the model in its split form:

$$\boldsymbol{\tau}_1 + \theta(\lambda)\overset{\nabla}{\boldsymbol{\tau}}_1 = \eta_s(\lambda)\dot{\boldsymbol{\gamma}} \tag{2}$$

$$\boldsymbol{\tau}_2 = \eta_\infty \dot{\boldsymbol{\gamma}} \tag{3}$$

$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 \tag{4}$$

In these equations, $\boldsymbol{\tau}$ is the extra stress tensor, i.e. it is the constitutively determined part of the total stress $\boldsymbol{T} = -\pi\mathbf{1} + \boldsymbol{\tau}$, where $\mathbf{1}$ is the unit tensor. The isotropic term $-\pi\mathbf{1}$ is determined by solving the balance equations together with their boundary conditions.[2] The tensor $\boldsymbol{\tau}_1$ is the viscoelastic stress, $\boldsymbol{\tau}_2$ is the viscous stress, $\theta(\lambda) \equiv \eta_s(\lambda)/G(\lambda)$ is the structural relaxation time, $\eta_s(\lambda)$ is the structural viscosity, η_∞ is the infinite-shear-rate viscosity (i.e. the viscosity of the material in its fully unstructured state), $G(\lambda)$ is the structural shear modulus, $\dot{\boldsymbol{\gamma}} \equiv \nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T$ is the rate-of-strain tensor field whose magnitude is $\dot{\gamma} \equiv \sqrt{\mathrm{tr}\,\dot{\boldsymbol{\gamma}}^2/2}$, $\boldsymbol{u}$ is the velocity vector field, and $\overset{\nabla}{\boldsymbol{\tau}}_1$ stands for the upper-convected time derivative of $\boldsymbol{\tau}_1$, given by

$$\overset{\nabla}{\boldsymbol{\tau}}_1 = \frac{d\boldsymbol{\tau}_1}{dt} - \boldsymbol{\tau}_1 \cdot (\nabla\boldsymbol{u}) - (\nabla\boldsymbol{u})^T \cdot \boldsymbol{\tau}_1 \tag{5}$$

where $d\boldsymbol{\tau}_1/dt \equiv \partial\boldsymbol{\tau}_1/\partial t + \boldsymbol{u} \cdot \nabla\boldsymbol{\tau}_1$ is the material time derivative of $\boldsymbol{\tau}_1$.

The structural viscosity $\eta_s(\lambda)$ is given by

$$\eta_s(\lambda) \equiv \eta_v(\lambda) - \eta_\infty = \eta_\infty\left[\left(\frac{\eta_0}{\eta_\infty}\right)^\lambda - 1\right] \tag{6}$$

where η_0 is the viscosity of the material in its fully structured state. In many cases of real materials η_0 is so large (maybe infinite) that it cannot be determined experimentally. Nevertheless, even in such cases a large but finite value of η_0 is in general a good approximation, and as far as numerical solutions are concerned, assuming a large finite η_0 is equivalent to a regularization, which is anyway required in most numerical schemes. $\eta_v(\lambda) \equiv \eta_\infty(\eta_0/\eta_\infty)^\lambda$ is the viscosity in the absence of elastic effects, i.e. it is the viscosity (corresponding to the structuring level λ) that would be observed in an experiment at fixed elastic strain ($\dot{\gamma}_e = 0$). More details are found in de Souza Mendes [15], de Souza Mendes [5] and de Souza Mendes and Thompson [13].

The structuring-level dependent shear modulus $G(\lambda)$ is assumed to be of the form

$$G(\lambda) = G_0 \exp\left[m\left(\frac{1}{\lambda} - 1\right)\right] \tag{7}$$

where G_0 represents the shear modulus of the material in its fully structured state and m a dimensionless positive constant that governs the sensitivity of G to the structuring level.

The coefficient of the upper-convected time derivative of $\boldsymbol{\tau}_1$ that appears in Eq. (2) is a characteristic time of the material that quantifies its elastic memory when the structuring level is λ:

$$\theta(\lambda) \equiv \frac{\eta_s(\lambda)}{G(\lambda)} = \frac{\eta_\infty\left[\left(\frac{\eta_0}{\eta_\infty}\right)^\lambda - 1\right]}{G_0 \exp\left[m\left(\frac{1}{\lambda} - 1\right)\right]} \tag{8}$$

It is worth noting that $\theta(\lambda)$ can be measured by means of the recently proposed QL-LAOS methodology [18].

To characterize the level of elasticity we adopt the time θ_0, defined as

$$\theta_0 \equiv \frac{\eta_s(1)}{G(1)} = \frac{\eta_0 - \eta_\infty}{G_0} \simeq \frac{\eta_0}{G_0} \tag{9}$$

where we used the fact that $\eta_\infty \ll \eta_0$ for all elasto-viscoplastic materials. That is, θ_0 is essentially the relaxation time of the material in its fully structured state. We combine Eqs. (8) and (9) to write $\theta(\lambda)$ in terms of θ_0 instead of G_0:

$$\theta(\lambda) \equiv \frac{\eta_s(\lambda)}{G(\lambda)} = \theta_0\left(\frac{\eta_\infty}{\eta_0 - \eta_\infty}\right)\frac{\left(\frac{\eta_0}{\eta_\infty}\right)^\lambda - 1}{\exp\left[m\left(\frac{1}{\lambda} - 1\right)\right]} \tag{10}$$

[2] Note that π is not the mechanical pressure $p \equiv -\frac{1}{3}\mathrm{tr}\,\boldsymbol{T}$, because $\boldsymbol{\tau}$ is not deviatoric. In fact, $\pi = p + \frac{1}{3}\mathrm{tr}\,\boldsymbol{\tau}$.