Composite Structures 183 (2018) 7–20

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Frequencies of FGM shells and annular plates by the methods of discrete singular convolution and differential quadrature methods

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ARTICLE INFO

Article history:
Received 8 November 2016
Accepted 16 November 2016
Available online 17 November 2016

Keywords:
FGM composites
Conical shells
Discrete singular convolution
Annular plates
Differential quadrature
Frequency

ABSTRACT

Free vibration analysis of curved structural components such as truncated conical shells, circular cylindrical shells and annular plates has been investigated numerically in this paper. The method of discrete singular convolution (DSC) and the method differential quadrature (DQ) are used for numerical simulations, respectively. Related partial differential equations governed the motion of the structures obtained from higher-order shear deformation theory have been solved by using these two methods in the space domain. Different material properties have been considered such as isotropic, laminated and functionally graded material (FGM). Four-parameter power law and simple power law distributions have been used for ceramic volume fraction in FGM cases. The numerical results related to free vibration of conical shells have obtained by the present two techniques compare well with the results available in the literature. Results for circular cylindrical shells and annular plates have also been presented for different geometric and material parameters. The effects of grid number and types of the grid distribution have also been investigated for annular plate and shells.

1. Introduction

Curved structural members such as conical shells and annular plate structures have wide applications in different engineering components such as aerospace, civil, petro-chemical and mechanical engineering and structural efficiency necessitates the vibration and buckling analyses for design. A number of detailed studies such as books, review papers and thesis had been published on modeling and analysis for plates and shells with various geometries [1–15].

Hutchinson presented an analytical solution for axisymmetric flexural vibrations of a thick free circular plate [16]. Natural frequencies of Mindlin circular plates have been detailed investigated by Irie et al. [17]. A key paper on natural frequencies of simply supported circular plates was published and some benchmark results are presented Leissa and Narita [18]. Differential quadrature (DQ) and harmonic differential quadrature (HDQ) methods have been used for circular plates by Civalek [19] and Civalek and Ülker [20]. Liew et al. [21] investigated static behavior of Mindlin plates on Winkler foundations via differential quadrature method. Detailed formulations and applications of harmonic differential quadrature method to structural components have been reported by Striz et al. [22]. An explicit formulation for obtaining the weighting coefficients is proposed by Shu and Hue [23]. Eight-node curvilinear differential quadrature formulations for thick plates have been presented and some results obtained for circular and annular plates by Han and Liew [24]. By using the mesh free method static and free vibration response of thin plates of complicated shape are analyzed by Lie and Chen [25]. Liu and Liew [26] have also used differential quadrature method for free vibration analysis of Mindlin sector plates. Free vibration analysis of curvilinear quadrilateral plates by the differential quadrature method were studied by Shu et al. [27]. A four-node coordinate transformation via discrete singular has been proposed for vibration problem of arbitrary straight-sided quadrilateral plates by Civalek [28]. Gordon and Hall [29] give detailed construction of curvilinear co-ordinate systems for mesh generation and plate problems. Two-dimensional orthogonal polynomials for vibration analysis of circular and elliptical plates have been concluded by Lam et al. [30]. Radial basis functions based meshless methods have been successfully applied to plate problems by Ferreira et al. [31–33]. Also different analytical and numerical methods have been used for analysis of plates and shell structures such as circular shells, conical shells and panels, circular and annular plates and rectangular plates [34–44]. In these studies, many of analytical and

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http://dx.doi.org/10.1016/j.compstruct.2016.11.051
0263-8223/

numerical methods such as; Rayleigh-Ritz, Galerkin, finite differences, boundary volumes, mixed and p-version or h-version of finite element methods, generalized or harmonic differential quadrature, reproducing kernels based meshless methods and Shannon or Lagrange kernel based singular convolution methods were applied for analysis and modeling of mechanical systems. Among these, finite elements, Ritz and differential quadrature methods have been used by many researchers.

In the present work, two approximate numerical solutions of the related governing equations for the free vibration analysis of shells and annular plates are presented. To achieve this, two numerical discretized approaches such as Shannon's delta based discrete singular convolution (DSC) and trigonometric test function based differential quadrature called harmonic differential quadrature methods (HDQ) have been used for numerical simulations of the systems differential equations for vibration. The obtained results are tested via literature results and the values obtained by FEM using the ANSYS packed program.

As far as these authors know it is the first time the methods of differential quadrature and discrete singular convolution have been applied and compared to the vibration problem of annular plates, conical and circular cylindrical shells. The rest of this paper is constructed as follows. Section two introduces the brief theory of differential quadrature method. Section three is devoted to methodology of discrete singular convolution and main equations have been given. In section four, fundamental equations have been given. Some convergence and parametric results are presented in section five. A conclusion is given at the end of the manuscript in section six.

2. Differential quadrature (DQ) method

In 1990s, the method of differential quadrature (DQ) was prosed for practical solution for the mathematical physics and engineering problems. Different types of problem in applied solid and fluid mechanics have modeled accurately by this method up to now [45–54]. A different quadrature approach called harmonic type of differential quadrature is used by Striz et al. [22]. In this type of DQ, the polynomial functions, for example power or Lagrange interpolated, and Legendre polynomials are not used as test functions. Instead, harmonic type of differential quadrature used the harmonic or trigonometric functions as trial polynomials. So, this approaches known as harmonic DQ method. Shu and Xue [23] firstly give an explicit form for the weighting coefficients in this type DQ. Let $f(x)$ is approximated by following series expansion [22]

$$f(x) = c_0 + \sum_{k=1}^{N/2}\left(c_k \cos\frac{k\pi x}{L} + d_k \sin\frac{k\pi x}{L}\right) \tag{1}$$

Lagrange interpolated trigonometric function is also defined [23]

$$h_k(x) = \frac{\sin\frac{(x-x_0)\pi}{2}\cdots\sin\frac{(x-x_{k-1})\pi}{2}\sin\frac{(x-x_{k+1})\pi}{2}\cdots\sin\frac{(x-x_N)\pi}{2}}{\sin\frac{(x_k-x_0)\pi}{2}\cdots\sin\frac{(x_k-x_{k-1})\pi}{2}\sin\frac{(x_k-x_{k+1})\pi}{2}\cdots\sin\frac{(x_k-x_N)\pi}{2}} \tag{2}$$

for $k = 0,1,2,\ldots,N$. Thus, the weighting coefficients in HDQ of the first-order derivatives A_{ij} for $i \neq j$ given as below explicit form:

$$A_{ij} = \frac{(\pi/2)P(x_i)}{P(x_j)\sin[(x_i - x_j)/2]\pi}; \quad i,j = 1,2,3,\ldots,N, \tag{3}$$

In above relation the P(x) is given as

$$P(x_i) = \prod_{j=1, j\neq i}^{N} \sin\left(\frac{x_i - x_j}{2}\pi\right); \text{ for } j = 1,2,3,\ldots,N. \tag{4}$$

As for higher-order derivatives for example second-order derivatives B_{ij} for $i \neq j$ the weighting elements are as follows [26]:

$$B_{ij} = A_{ij}\left[2A_{ii}^{(1)} - \pi ctg\left(\frac{x_i - x_j}{2}\right)\pi\right]; \quad i,j = 1,2,3,\ldots,N, \tag{5}$$

For diagonal elements following relation can be used

$$A_{ii}^{(p)} = -\sum_{j=1, j\neq i}^{N} A_{ij}^{(p)}; \; p = 1 \text{ or } 2; \text{ and for } i = 1,2,\ldots,N. \tag{6}$$

2.1. Choices of sampling grid points

A significant issue on convergence and efficiency of the numerical methods is the choice of the grid numbers. Also, sampling of these grid points have important. It is know that different types of sampling points in related coordinate directions are possible. In general, two approaches have been widely used for grid sampling. A well-known approach is equally sampling grids (ESG-1):

$$x_i = \frac{i-1}{N-1} \tag{7}$$

Another, as second is known as the non-equally grid points (NESG-1) in the space directions;

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\right)\pi\right] \tag{8}$$

For annular plates the following two grid point such as equally spaced sampling (ESG-2) and non-equally spaced (NESG-2) sampling grids has also been used for inner points in literature as below:

$$x_i = r_i = R_1 + (R_2 - R_1)\left[\frac{i-2}{N-3}\right] \tag{9}$$

$$x_i = r_i = R_1 + (R_2 - R_1)\left[1 - \cos\frac{(i-2)\pi}{N-3}\right] \tag{10}$$

where R_1 and R_2 are inner and outer radius of annular plates, respectively.

3. Discrete singular convolution (DSC)

This novel numerical method called the discrete singular convolutions (DSC) has formulated by Wei [55]. This method is also very simple in applications and programming and highly efficient in smoothed problems. This method based on some transforms in mathematics such as the Hilbert, Abel and Radon transforms. It is also stated that the origin of this method called DSC is the theory of distributions. In fact the wavelet theory is also related directly with this method [56–62]. The wavelets theory can found application area in telecommunication and electromagnetic theory. In this comparative study, expended formulations related to the method of DSC not presented. Up to now, researcher has been used of this method for numerical solution of applied mechanics in plates and shells problems and problems related to computational fluid dynamics [63–66,28,67–69]. Some related papers on frequency response, stability and stress analysis of rods, truss, frames, beams, plates and panels via DSC are found in literature [70–77]. From these papers it is not wrong to say that the method of DSC is very practice and results have good accuracy for macro and micro/nano-scaled structures such as beams, nano-plates and nano structures [78–87]. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [55]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{11}$$

The known kernel is $T(t-x)$ and it is singular form in above equation. The method of singular convolution is more useful via some approximation kernels. One of the best is the regularized Shannon kernel. This kernel written as [56]