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Bending and buckling of a metal seven-layer beam with crosswise corrugated main core – Comparative analysis with sandwich beam

E. Magnucka-Blandzi

Institute of Mathematics, Poznan University of Technology, ul. Piotrowo 3A, 60-965 Poznań, Poland

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ABSTRACT

The study is devoted to thin-walled simply supported sandwich beams: three and seven-layer beams. Seven-layer beams are composed of a trapezoidally crosswise corrugated main core and two three-layer facings with trapezoidally lengthwise corrugated cores. Essential difference between such a structure and classical three-layer beams consists in the fact that the facings are of three-layer structure too. Hence, the structure is a seven-layer beam, the layers of which are distinguished by different properties. The thin-walled beam with three trapezoidally corrugated cores is an innovatory orthotropic structure, not referred to in the literature. Characteristic feature of the beam consists in differentiation of shear effects in particular layers, according to the core corrugation direction. Deformation of the cross section of the beam depends on the direction too. An original mathematical model is formulated, which includes the hypothesis of deformation of the cross section of the beam, inclusive of the displacement and strain fields, and rigidities of the layers in particular directions. The adequate model of three-layer beam with steel foam core is also formulated. Basing on the principle of the total potential energy the equations of equilibrium are derived for three and seven-layer beams, respectively. Then the equations are analytically solved. In case of seven-layer beams the deflections and the critical axial force are determined for different values of the trapezoidal corrugation pitch of the main core. The results are compared with adequate for three-layer beam and presented in Tables. It is concluded that such seven-layer beam – plate band with three-layered facings is more resistant to the load than the classical three-layered beams of the same mass with regard to strength and stability. Additional effect of the research consists in generalization of classical theories of sandwich plates.

1. Introduction

Sandwich structures are widely applied in mechanical and civil engineering since mid-twentieth. During the last 5 decades the properties of sandwich structures composed of a light core and two thin facings have been extensively studied. The first basic article on the sandwich plates with a corrugated core was written by Libove et al. [1] in 1951 and first monograph devoted to the analysis and design of structural sandwich panels was written by Allen [2] in 1969. Ashby et al. [3] are the authors of a design guide of metal foams. Review of Zig-Zag theories for multi-layered plates and shells was presented by Carrera [4] in 2003. Different hypotheses (theories) of sandwich plates were compared in the another review article by Carrera and Brischetto [5]. Whereas applications, manufacturing and material properties of the steel foam for structures were reviewed by Smith et al. [6] in 2012. The theoretical study of transverse shear modulus of elasticity for thin-walled corrugated cores of sandwich beams was presented by Isaksson et al. [7] in 2007, Magnucka-Blandzi and Magnucki [8] in 2014, Lewinski et al. [9] in 2015, and Magnucka-Blandzi et al. [10] in 2015. The buckling and post-buckling problems of an elastic seven-layered cylindrical shell under uniformly distributed pressure was analyzed numerically by Malinowski et al. [11] in 2015. He et al. [12] investigated the low velocity impact behavior of hybrid sandwich structures consisting of carbon fiber reinforced polymer face sheets and aluminum alloy corrugated cores by experimental and numerical method in 2016. The same year the effects of geometric parameters and laminate construction on the global bending response of composite sandwich plates with corrugated core were determined by Boorle and Mallick [13,14].

The subject of the paper is the metal seven-layer beam – plate band. Experimental investigations and the finite elements analysis of these beams were presented by Paczos et al. [15] in 2016. Preliminary studies were devoted to five-layered beams [16–18]. The theoretical, experimental, and numerical investigations were conducted. These beams consisted of two-layered facings and the

E-mail address: ewa.magnucka-blandzi@put.poznan.pl

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main core. The outer layers were flat. The inner layers of facings were trapezoidally corrugated, as well as the main core (but in the perpendicular direction to the facings corrugation).

The paper is devoted to the seven-layered beam (the band plate) with lengthwise corrugated inner layers of facings and crosswise corrugated main core (Fig. 1). The outer layers of facings are flat. The beam is compared to the classical three-layer beam with the core made of a steel foam (Fig. 2). The outer layers are also flat.

The same mass of these beams is assumed. It is presented that such seven-layer beam is more resistant for bending and buckling than the classical sandwich beam. The beams are simply supported. Two cases of loading are taken into account. In the first case beams are subjected to an axial force, in the second case beams are subjected to three-point bending.

2. Analytical description of the seven-layer beam

2.1. The field of displacements

Based on the papers [19] the displacements of the cross section of the beam are assumed as it is presented in Fig. 3. The shear effect occures in a core of facings.

The longitudinal displacements v are defined as follows (according to the hypothesis presented in Fig. 3):

- the outer flat sheets (the upper: the sign "−", the lower: the sign "+"):

$$v(y,z) = -z\frac{dw}{dy} \mp v_1(y) \tag{1}$$

for $-\left(\frac{1}{2}t_{c1} + 2t_s + t_{c2}\right) \leqslant z \leqslant -\left(\frac{1}{2}t_{c1} + t_s + t_{c2}\right)$ and $\frac{1}{2}t_{c1} + t_s + t_{c2} \leqslant \frac{1}{2}t_{c1} + 2t_s + t_{c2}$ respectively,

- the lengthwise corrugated cores of facings (the upper: the sign "+", the lower: the sign "−"):

$$v(y,z) = -z\frac{dw}{dy} + \left[z \pm t_{c1}\left(\frac{1}{2} + x_1\right)\right]\phi(y) \tag{2}$$

for $-\left(\frac{1}{2}t_{c1} + t_s + t_{c2}\right) \leqslant z \leqslant -\left(\frac{1}{2}t_{c1} + t_s\right)$ and $\frac{1}{2}t_{c1} + t_s \leqslant z \leqslant \frac{1}{2}t_{c1} + t_s + t_{c2}$ respectively,

- the inner flat sheets and the crosswise corrugated main core:

$$v(y,z) = -z\frac{dw}{dy}, \tag{3}$$

for $-\left(\frac{1}{2}t_{c1} + t_s\right) \leqslant z \leqslant \frac{1}{2}t_{c1} + t_s$,

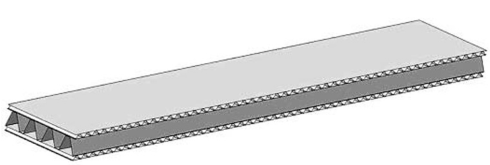

Fig. 1. Seven-layer beam with trapezoidal corrugated cores.

Fig. 2. Three-layer beam with a metal foam core.

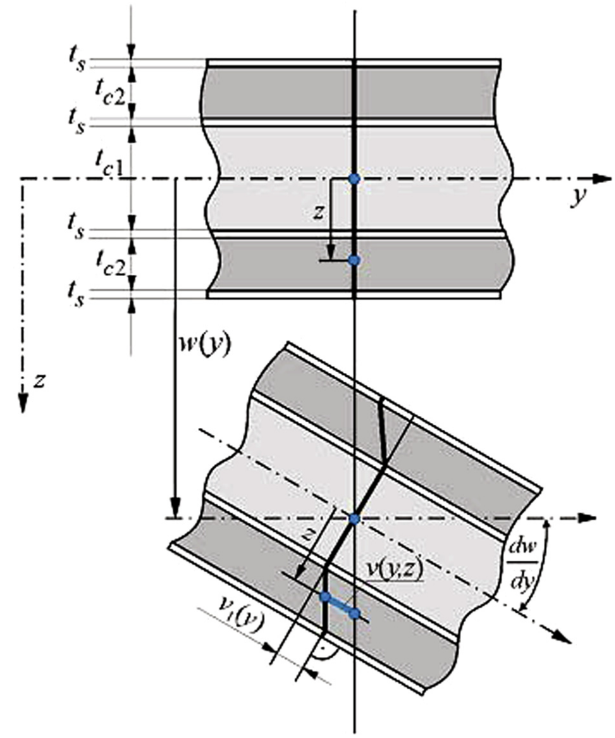

Fig. 3. Scheme of displacements field.

where w – deflection, $\phi(y) = v_1(y)/t_{c2}$ – dimensionless function determines the field of displacement, t_{c1}, t_{c2}, t_s – thicknesses of layers (Fig. 3) and denotations – indexes $c1$ – main core, $c2$ – inner corrugated layer of facings, s – sheets.

2.2. Strains and stresses

Normal and shear strains in all layers of the beam are defined as follows

$$\varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{dw}{dy}. \tag{4}$$

So, the shear strains are not equal to 0 in the corrugated cores of facings, and

$$\gamma_{yz}^{(c2)} = \phi(y). \tag{5}$$

The layers are distinguished by different properties and rigidities in particular directions. The mechanical properties of the individual layers are described in [9], and in particular, both the shear modulus and Young's modulus.

Normal and shear stresses are defined according to Hooke's law.

- for the flat sheets and the crosswise corrugated main core

$$\sigma_y^{(s)} = E\varepsilon_y^{(s)}, \quad \tau_{yz}^{(s)} = 0, \quad \sigma_y^{(c1)} = E_y^{(c1)}\varepsilon_y^{(c1)}, \quad \tau_{yz}^{(c1)} = 0, \tag{6}$$

- for the lengthwise corrugated cores of facings

$$\tau_{yz}^{(c2)} = G_{yz}^{(c2)}\gamma_{yz}^{(c2)}, \quad \sigma_y^{(c2)} = E_y^{(c2)}\varepsilon_y^{(c2)}, \tag{7}$$

where E – the Young's modulus of metal sheets, $E_y^{(c1)}, E_y^{(c2)}$ – the Young's modulus of the corrugated main core and the facings core respectively presented in [19], $G_{yz}^{(c2)}$ – the shear modulus of the facings core introduced in [9].

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