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# Influence of hydrodynamic drag model on shear stress in the simulation of magnetorheological fluids

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ABSTRACT

Simulations of magnetorheological fluids are performed with different models for the hydrodynamic drag law. The shear stress predictions from two coupled discrete element – smoothed particle hydrodynamics models with different drag laws are compared to pure discrete element simulations for a wide range of Mason numbers. The discrete element model has a higher computational efficiency but the treatment of the hydrodynamic drag force involves some rough approximations. Based on the results of this study, a criterion is proposed for the applicability of the pure discrete element model in the simulation of sheared magnetorheological suspensions.



## 1. Introduction

In many particle-based simulations of magnetorheological fluids (MRF), the Stokes drag law is used to treat the interaction between the fluid and the particles [1–6]. However, it is only strictly valid for very dilute suspensions which is usually not the case in a magnetorheological fluid. Furthermore, often a one-way coupling is used for the simulations, i.e. the velocity of the fluid phase is not influenced by the presence of the particles in the suspension [1–6]. Thus, the treatment of the particle–fluid interaction is approximated in two ways.

To account more accurately for the particle–fluid interaction, several different approaches are found in MRF literature. Gao et al. [7] proposed a hydrodynamic interaction tensor to account for the influence of suspended particles on the fluid motion. A two-way coupled model has been used by Kang et al. [8] to study single particle chains where the magnetic interaction was treated with the discrete element method (DEM) and the hydrodynamics were computed with the finite element method. Denser suspensions have been studied using a two-way coupled DEM-Lattice Boltzmann approach [9]. However, comparison to the one-way coupled DEM model is only provided on the basis of exact trajectories for systems consisting of a single particle chain [7], or not at all [8,9].

In this work, we will address the question under which conditions the one-way coupled DEM model is sufficient to capture the MRF behavior correctly and for which cases, on the contrary, the particle–fluid coupling is necessary. For the coupled simulations, a DEM-smoothed particle hydrodynamics (SPH) approach is used. Instead of focussing on the exact positions of single particles, the shear stress in dense suspensions is studied which is a relevant quantity for many MRF applications.

Furthermore, the influence of the specific drag law will be investigated. While the Stokes drag law is widely accepted for the use in dilute suspensions, it considerably underestimates the drag force at higher volume fractions. A phenomenological model for the drag force in denser suspension was proposed by Dallavalle and Di Felice [10,11], which has been successfully applied to the simulation of fluidized beds [12,13]. To investigate the influence of the drag law, the DEM–SPH coupled simulations have been performed with the Stokes model as well as with the Dallavalle/Di Felice model.

The simulation results are compared between different models and to experiments. The aim is to derive an easy-to-check criterion

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for the applicability of the one-way coupled DEM model with Stokes drag law. As this is the most widely used particle-based simulation approach in MRF literature, such a criterion would be a major benefit for all further research in this field.

The paper is organized as follows. In Section 2, the simulation methods are described. In Section 3.1, shear stresses from simulations are compared to experimental results and theoretical predictions. In Section 3.2, differences between the simulation models are shown. The role of hydrodynamics in the simulations is discussed. As a result, a criterion for the applicability of the one-way coupling is proposed. The results are summarized in Section 4.

## 2. Simulation setup and numerical method

### 2.1. Simulation setup

For the present study, simulations of an MRF shear cell are performed. The shear cell is modeled as a three-dimensional simulation box, terminated by two parallel solid walls in $x$-direction and with periodic boundary conditions in $y$- and $z$-direction. The MRF is confined between these walls. The numerical description of the shear cell walls and the magnetic particles in the MRF is based on the discrete element method. The walls are modeled as ensembles of overlapping, nonmagnetic particles. The iron particles in the MRF are represented as monodisperse magnetized spheres. The radius of wall particles and magnetic particles is $R = 2.5\ \mu m$. A particle volume fraction of $\phi = 30\%$ was chosen for all simulations which is a typical volume fraction for many MRF applications [14]. Initially, the magnetic particles are located at random positions between the two moving walls. The gap height between the upper and lower wall is $L_x = 150\ \mu m$, i.e. the centers of the wall particles are positioned at $x^{u,l} = \pm 75\ \mu m$. The corresponding wall velocities are given by $\mathbf{v}^u = (0, 0, x^u\dot{\gamma})$ for the upper wall and $\mathbf{v}^l = (0, 0, x^l\dot{\gamma})$ for the lower wall, where $\dot{\gamma}$ is the applied shear rate. The magnetizations of the particles are pointing in $x$-direction. The lengths of the simulation box in the periodic directions are $L_y = 50\ \mu m$ and $L_z = 120\ \mu m$. The simulation setup is shown in Fig. 1. In the case of coupled DEM–SPH simulations, additional SPH walls are present at the same location as the DEM walls. The carrier fluid represented by SPH particles is confined between the walls and interacts with the suspended DEM particles via coupling forces.

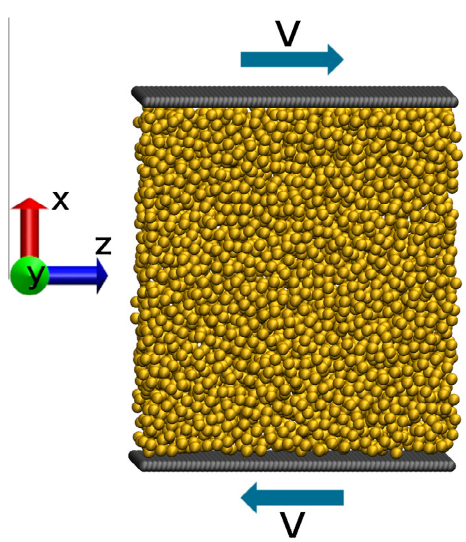


**Fig. 1.** Simulation setup of the MRF shear cell. The walls are modeled as an ensemble of overlapping spherical particles. They are moved in $z$-direction with velocities $\pm\mathbf{v}$. Confined between the walls are the randomly distributed magnetic particles. The carrier fluid is not displayed.

### 2.2. Numerical method

To investigate the influence of the drag force implementation on the shear stress, simulations have been performed with three different drag models:

1. One-way coupled DEM simulations with Stokes drag law for the particle–fluid interaction (referred to as *DEM-Stokes*).
2. Two-way coupled DEM–SPH simulations with Stokes drag law for the particle–fluid interaction (referred to as *DEMSPH-Stokes*).
3. Two-way coupled DEM–SPH simulations with the Dallavalle/Di Felice drag law for the particle–fluid interaction (referred to as *DEMSPH-DD*).

The simulations presented in this paper have been performed with the SimPARTIX® software package [15]. Note that the computational effort for the coupled DEM–SPH models is around 10 times larger compared to the pure DEM model.

In the following, the different simulation models are described.

#### 2.2.1. Discrete element model

The forces included in the discrete element model are magnetic interaction forces between the magnetic particles, normal elastic Hertzian repulsion [16], and a drag force exerted by the fluid on the particles. Gravity and Brownian forces, as well as elastic-frictional shear between particles, are neglected. Forces are only computed for the freely moving DEM particles, but not for the shear cell walls. The wall movement is determined by a prescribed velocity and is independent of the forces on the wall particles. The motion of a DEM particle $i$ is described by

$$m_i \frac{d\mathbf{v}_i}{dt} = \sum_j \mathbf{F}_{ij}^{\text{cont}} + \sum_k \mathbf{F}_{ik}^{\text{mag}} + \mathbf{F}_i^{\text{hyd}}, \tag{1}$$

with $m_i$ and $\mathbf{v}_i$ as mass and velocity of particle $i$, $\mathbf{F}_{ij}^{\text{mag}}$ and $\mathbf{F}_{ij}^{\text{cont}}$ as the magnetic and contact forces exerted by particle $j$ on particle $i$ and $\mathbf{F}_i^{\text{hyd}}$ as the drag force of the fluid on the particle. The sum $\sum_j$ runs over all particles in direct contact with particle $i$. The sum $\sum_k$ runs over all magnetic interaction partners of particle $i$.

For the repulsive contact force between two particles, often an exponential repulsion law is used (see e.g. [1–4,17]). For the present work, the physically motivated Hertzian repulsion law was chosen instead. A comparison between the Hertzian repulsion law and the traditionally used exponential repulsion showed that both models should be equally suited for the use in MRF simulations [18]. The Hertzian contact force of particle $j$ on particle $i$ is given by

$$\mathbf{F}_{ij}^{\text{cont}} = \left( \frac{\frac{2}{3}Y}{1 - \nu^2} \sqrt{R_{\text{eff}}} h_{ij}^{3/2} \right) \frac{\mathbf{r}_{ij}}{r_{ij}}, \tag{2}$$

with $Y$ as Young's modulus, $\nu$ as Poissons's ratio, $R_i,\ R_j$ as the radii of the particles $i$ and $j$, $R_{\text{eff}} = R_iR_j/(R_i + R_j)$ as an effective particle radius and $h_{ij} = \max\{R_i + R_j - |\mathbf{r}_{ij}|, 0\}$ as the particle overlap [16].

The magnetic force between two particles $i$ and $k$ with magnetic moments $\mathbf{m}_i$ and $\mathbf{m}_k$ is given by

$$\mathbf{F}_{ik}^{\text{mag}} = \frac{3\mu_0}{4\pi} \left[ \frac{(\mathbf{m}_i \cdot \mathbf{m}_k)\mathbf{r}_{ik} + (\mathbf{m}_k \cdot \mathbf{r}_{ik})\mathbf{m}_i + (\mathbf{m}_i \cdot \mathbf{r}_{ik})\mathbf{m}_k}{r_{ik}^5} - 5\frac{(\mathbf{m}_i \cdot \mathbf{r}_{ik})(\mathbf{m}_k \cdot \mathbf{r}_{ik})\mathbf{r}_{ik}}{r_{ik}^7} \right], \tag{3}$$