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Wave propagation in pre-deformed periodic network materials based on large strains homogenization

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A B S T R A C T

This paper explores the influence of large deformations on the propagation of acoustic waves in repetitive network materials. The problem of elastic wave propagation in pre-deformed elastic materials and structures is highly interesting in many applications. Both theoretical and numerical methods are developed in this contribution in order to assess the influence of finite strains developing within repetitive networks on the evolution of their band diagrams. An incremental scheme for the update of frequency and phase velocity of the computed homogenized medium has been developed successively considering 1D and 2D structures; it incorporates an update of the frequency and phase velocity of the propagating waves versus the effective density and the state of finite deformation of the effective continuum used as a substitution medium for the initial repetitive network. The applied deformation is shown to have significant effects on the wave frequency and phase velocity. Especially, it is shown that the phase velocity for the hexagonal network strongly decreases under finite compressive strains. The influence of the effective density on the dispersion relation and band diagrams under the application of an incremental deformation over the lattice unit cell is shown.

1. Introduction

Structures having a periodical distribution of the geometry and material properties of their constituents present interesting wave propagation properties, like the existence of frequency band gaps, local resonances, and response directionality due to their anisotropy, left-handedness, cloaking, or negative acoustic refraction. These unusual acoustic properties are due to material and structural heterogeneities associated to periodic modulations of the stiffness and inertial properties, resulting e.g. from modifications of the microstructural configuration. Moreover, the field of acoustic metamaterials has raised a considerable interest due to the possibility of tailoring their microstructure to obtain various interesting effects like local resonances, partial or full band gaps, see [12] and references therein. Soft metamaterials have the capability to sustain large deformations, and as a consequence they offer promising opportunities to adjust the acoustic characteristics through the deformation.

Many techniques have been developed to predict the mechanical properties of heterogeneous structures, and especially network materials, thereby bypassing the need to resolve the smallest spatial scale [7]. Network materials find applications at different scales, and their multiscale feature makes them especially interesting as reinforcements for composite materials, regarding both their static and dynamic properties, as exposed in the recent special issue [10].

Homogenization methods aim at describing the overall response of heterogeneous structures including composites and periodic structural lattices in terms of their effective properties, as presented in the recent contribution [46] and references therein. More recently, [19] extended the linear model developed in [17,18] to construct the stress–strain relation and the strain energy function for hyperelastic cellular materials with arbitrary symmetry. An alternative approach was proposed in [46] using a computational homogenization method to derive a nonlinear constitutive model for network materials.

The incorporation of nonlinear aspects of wave propagation in structures is necessary whenever large deformations occur [3,30,28,32], but remains a considerable challenge. Two types of nonlinearities may be present in a broad sense, namely material nonlinearities [23,24] and geometrical nonlinearities [31]. This last type of nonlinearity is related to the evolution of the microstructure or structure configuration, for instance the change of configuration of a repetitive network, and it can be modeled as a succession of incremental deformations associated to the modification of the structure geometry [31]. The presence of a nonlinearity in periodic structures results in amplitude-wave dependency in the dispersion relations; this opens new

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possibilities for a passive tuning of the dispersion band structure through an amplitude-dependency of propagating waves, thereby going beyond a mere control of the dynamic and acoustic properties of repetitive structures by the design [24] or by the application of an external stimulus.

A nonlinear periodic structure supports a variety of wave solutions depending on wave amplitude, waves interactions, and type of nonlinearity, for example solitary wave solutions for Boussinesq type equation, harmonic plane wave and discrete breathers [5,39,42–45,40,33,34]. In [1], the authors analyze the dispersion relations for axial and flexural elastic wave motion in homogeneous beams subjected to finite strains. In [21], the authors study wave dispersion in a one-dimensional nonlinear elastic metamaterial; the large elastic deformation provides the nonlinearity in the thin rod, whereas the metamaterial behavior is associated with the dynamics of the local resonators. [47] developed a numerical analysis improved by experimental measures to show the evolution of the locally resonant band gap under a nonlinear pre-deformation. [2] demonstrate the ability to use deformation to transform phononic band gaps in periodic elastomeric structures. In [25] the authors focus on materials which constitutive law contains a cubic stress–strain nonlinearity, while [4] present an analytical and numerical method to describe the propagation of nonlinear waves within a structure endowed with a square stress–strain relationship.

The effect of pre-stress or pre-strain on wave propagation through homogeneous anisotropic media has raised the interest of many authors; the effect of preexisting finite elastic deformations on wave propagation has been analyzed in [13,30] and the effects of incremental deformations on a homogeneous continuum medium has been studied in [11,6]. The initial deformation must be large enough to change the geometry of the medium, since an infinitesimal initial deformation would not affect the properties of the material based on the superposition principle valid for small deformations. The incremental effective properties of pre-stressed homogeneous media undergoing large deformation have been analyzed in [37,38], wherein the authors put some restrictions on the strain energy function for elastic waves to be able to propagate within the material.

In [20,22,27,29,36], the authors analyze the propagation of waves in composites consisting of a small number of layers (two or three layers) undergoing sufficiently large deformations.

It is worth mentioning that studies of nonlinear wave propagation in structures essentially deal with one-dimensional (1D) systems, whereas the nonlinear wave dynamics in multi-dimensional (2D and 3D) discrete systems has not been thoroughly investigated so far.

In this paper, we analyze the dynamical properties of periodic network materials subjected to finite strains, relying on dedicated homogenization techniques developed to substitute the initial discrete periodic lattice by an effective Cauchy continuum (relying on Bernoulli beams). An incremental scheme for the update of network geometry, mechanical response and frequency is set up successively in 1D and 2D situations, based on the effective nonlinear medium obtained by homogenization.

Novel aspects presented in this paper are the following:

- An incremental scheme for the update of the frequency of the homogenized medium is developed successively considering 1D and 2D network materials, based on the update of the tangent stiffness matrix of the homogenized continuum;
- We study the influence of the geometrical nonlinearities developed within the networks due to the imposed finite strains on the dispersion relations and band diagrams;
- We develop numerical schemes to simulate the influence of finite strains for 2D repetitive lattice materials, illustrated by networks which become auxetic under a kinematic control.

The incremental homogenization method at the basis of the dynamical analysis is developed in Section 2. We analyze in Section 3 the wave propagation in a one-dimensional context, representative of a macroscopic beam incorporating a repetitive microstructure submitted to pure tension. An extension of the method to the determination of the incremental dispersion relations for 2D periodic structures is done in Section 4, which demonstrates the impact of finite strains on partial band gaps and phase velocity. We conclude this work in Section 5 with a synthesis of the main effects of the imposed deformation on nonlinear wave propagation.

Regarding notations, vectors and second order tensors are denoted with boldface symbols. The transpose of a second order tensor $\mathbf{A}$ is the second order tensor noted $\mathbf{A}^T$. The summation convention on repeated indices is presently adopted, otherwise explicitly stated. The second order identity tensor is denoted $\mathbf{I}$.

2. Microscopic and mesoscopic nonlinear homogenization problems

The adopted computational method of the effective nonlinear response of periodic network materials relies on a two steps methodology: the ground state effective moduli are first evaluated in the initial small strains regime, followed by the evaluation of the subsequent nonlinear response, based on the update of the lattice configuration (geometry) when subjected to an increased kinematic loading imposed over the identified network unit cell. We rely for the purpose of computing the effective nonlinear response on the discrete homogenization method (abbreviated DH method in the sequel) to replace the initially discrete structure by a nonlinear elastic effective continuum.

2.1. Computation of the small perturbations homogenized elastic response of the network

The general idea at the base of the method is the periodic repetition of an elementary cell made of beams connected at nodes to define an infinite network. Consider a finite 2D (surface) or 3D structure, parameterized by a small parameter ε, defined as the ratio between a characteristic length of the lattice unit cell to a characteristic length of the entire network, scalar quantity L (Fig. 1). For a large enough

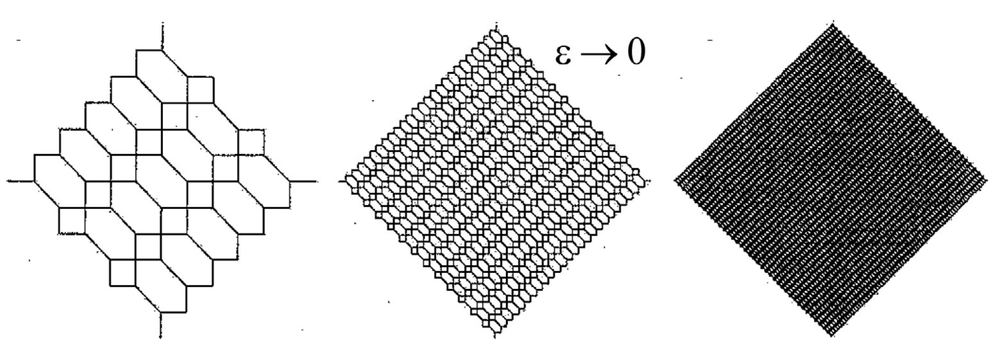

Fig. 1. Set of repetitive lattices parameterized by a small parameter ε.