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Electrical resistance change vs damage state in cracked symmetric laminates: A closed form solution

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A B S T R A C T

An analytical solution is developed to assess the stiffness loss in conductive symmetric laminates, caused by the presence of off-axis cracks, as a function of the intrinsic electrical resistance change.

In order to achieve this ambitious aim, initially the attention is focused on the electrical problem, and a closed form solution is developed which allows the electrical resistance of the laminate to be estimated starting from the density of transverse cracks. Such an expression is later inverted and used in combination with a new analytical mechanical model suitable to estimate the stiffness degradation associated to a given crack density.

The accuracy of the proposed solutions is verified by comparison to a bulk of FE analyses, also discussing the effect of the most influencing material and geometrical parameters.

1. Introduction

In the frame of advanced structural applications, the development of methods to provide real-time information about the damage state of composite parts has become a crucial issue. This process, that includes the use of sensors permanently mounted on the structure, is generally referred to as Structural Health Monitoring (SHM). SHM can lead to several benefits, especially in the case of composites, increasing the reliability and reducing the costs of inspection and maintenance of engineering structures.

The damage evolution of a composite part subjected to static or fatigue loads is generally associated to the onset and propagation of matrix cracks, delamination between the plies and fibre breakage. In particular, the onset of off-axis cracks is typical of the first stages of damage and, despite their presence does not substantially compromise the load bearing capability of the structure, it leads to a decrease of the stiffness, which sometimes is unacceptable, and might compromise its functionality. It has been shown in the literature that electrical-based methods can be effectively used to monitor the presence of delaminations and fibre breakage in CFRPs, enabling the detection of fatigue and impacts-induced damage [1–3]. One of the conditions that limited, so far, the use of electric potential-based methods is that their applicability is restricted only to composites made of conductive fibers. However, the recent advances in the field of nanotechnology allowed nanoparticles to be incorporated within traditional composites to obtain multifunctional materials with enhanced mechanical and physical properties. Even if, by one hand side, the dispersion of CNTs continues to be a challenging issue, the development of scalable dispersion techniques [4,5], and the commercial availability of pre-dispersed master-batches have made it possible the effective nanomodification of conventional composites. In particular, many researchers reported that the nanomodification with conductive particles, such as carbon nanotubes (CNTs), can provide conventional polymers with electrical conductivity, creating a conductive network within the material already at very low weight fractions [6,7]. The onset of matrix cracks and delamination causes the rupture of CNT paths, resulting in an irreversible increase of the electrical resistance of the composite. Different from other SHM techniques, in this case the damage is associated to a variation of a bulk property of the material (i.e., the electrical resistance), making it self-sensing to the presence of damage. A direct relationship between the electrical resistance change of CNT/glass/epoxy composites and their damage state was experimentally documented by several authors. Gao et al. [8] tested cross-ply coupons in FRP made of CNT modified matrix, and noted an irreversible increase of the electrical resistance as the crack density increased. Thostenson and Chou [9] demonstrated that GFRPs modified with multi-walled CNTs can be used as self-sensing materials to evaluate the onset and evolution of the matrix-dominated failure at the initial stages of damage. Böger et al. [10] performed fatigue tests on multidirectional CNT/glass/epoxy laminates showing that the decrease of the dynamic modulus of the material due to damage evolution is associated to an irreversible increase of the electrical resistance. Despite these promising experimental results, so far only limited efforts have been devoted to the development of analytical or numerical models to predict the electrical resistance change of a damaged laminate as a

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Nomenclature

θ_1, θ_2	orientation of the plies
h_1, h_2	thickness of the plies
L	distance between two adjacent cracks
ρ	crack density
I_x	global electric current injected
$x_1^{(2)}$, $x_2^{(2)}$, z_i	coordinates of the cartesian system aligned to θ_2
$e_{x1}^{(i)}$, $e_{x2}^{(i)}$	electric field of a θ_i lamina with respect to the coordinate system aligned to θ_2
$V^{(1)}$, $V^{(2)}$	electric potential
$\overline{V}^{(1)}$, $\overline{V}^{(2)}$	averaged value of the electric potential along the thickness
η_1, η_2, η_z	electrical resistivity in the material coordinate system
$\Gamma_{kj}^{(i)}$	term of the resistivity matrix of a θ_i lamina with respect to the coordinate system aligned to θ_2
$j_1^{(i)}$, $j_2^{(i)}$, $j_z^{(i)}$	current density of a θ_i lamina with respect to the co-ordinate system aligned to θ_2
j_{int}	interface current density
$\bar{j}_k^{(i)}$	current density averaged through the thickness
$\tilde{j}_2^{(2)}$	current density flowing at the undamaged state
φ	coefficient to account for the electrical bridging across cracks
R_x	electrical resistance
R_{x0}	electrical resistance of the laminate without cracks
Q_{kj}	term of the stiffness matrix in the material coordinate system
$\hat{Q}_{kj}^{(i)}$	term of the stiffness matrix of a θ_i lamina with respect to the coordinate system aligned to θ_2
$\hat{S}_{kj}^{(i)}$	term of the compliance matrix of a θ_i lamina with respect to the coordinate system aligned to θ_2
$u_1^{(i)}$,$u_2^{(i)}$	displacement of a θ_i lamina along $x_1^{(2)}$, $x_2^{(2)}$ directions
$\bar{u}_1^{(i)}$,$\bar{u}_2^{(i)}$	averaged value of the displacement along the thickness
$\sigma_{kj}^{(i)}$	stress component on a θ_i lamina expressed in the co-ordinate system aligned to θ_2
$\varepsilon_{kj}^{(i)}$	strain component on a θ_i lamina expressed in the co-ordinate system aligned to θ_2
	interface stresses
σ_{xg}	global stress applied to the laminate
E_x	longitudinal modulus
E_{x0}	longitudinal modulus of the laminate without cracks

function of the damage state. Li and Chou [11] computationally modelled the nanotube network embedded in fibre composites for damage sensing, finding that the technique is capable of detecting the onset of damage.

More recently, Gallo and Thostenson [12], investigated the sensitivity of CNT/glass/epoxy composites to the presence of matrix cracks in terms of electrical resistance change, taking advantage of experimental and numerical analyses.

Carraro et al. [13], dealing with a symmetric cross-ply laminate made of a conductive matrix, proposed a closed form solution directly correlating the stiffness degradation and the Direct Current (DC) electrical resistance change caused by a certain amount of transverse cracks.

Even if highly desirable for the effective engineering application of electrical-based methods for the health monitoring of composite parts, in the best of the authors' knowledge, a similar solution does not exist for layups different from $[0_n/90_m]_S$.

As a first step to fill this gap, in the present work the case of a $[(\theta_1)_n/(\theta_2)_m]_S$ laminate, where θ_1 and θ_2 are two generic orientations, is studied, and a closed form solution is obtained to directly correlate the electrical resistance change and the stiffness drop of the laminate, both due to a certain crack density in the θ_2 layers. To this end, initially, an analytical solution is developed to link the DC electrical resistance of the cracked laminate to the density of transverse cracks. Independently, taking advantage of an optimal shear lag analysis, a mechanical model was developed which allows the laminate stiffness degradation to be estimated as a function of the crack density. In this way, an analytical expression capable of predicting the laminate stiffness degradation as a function of the laminate resistance increase is obtained.

The accuracy of the proposed analytical solution is verified by comparison to a large bulk of FE analyses, and a parametrical analysis is carried out in order to highlight the effect of the most influencing material and geometrical parameters.

2. Evaluation of the electrical resistance as a function of the crack density

2.1. Statement of the problem

Let us consider a $[(\theta_1)_n/(\theta_2)_m]_S$ laminate made of unidirectional plies, and suppose that the layers with orientation θ_2 are cracked. The thickness of the plies with an orientation angle θ_1 and θ_2 are h_1 and h_2, respectively. Suppose that a direct current I_x is injected through the surfaces of the laminate, as shown in the schematic of Fig. 1(a), where, taking advantage of the symmetry of the problem, only the upper half of the laminate is represented.

In order to simplify the mathematical description of the problem under investigation, the following damage morphology is assumed:

- Cracks are parallel to the fibre direction in the θ_2 layers;
- Cracks are uniformly spaced, with a distance L (see Fig. 1b), and involve the whole width of the laminate. Thus, the crack density can be estimated as $\rho = 1/L$.

Thanks to these simplifications, the analysis can be restricted to a representative volume element (RVE) of material comprised between two adjacent cracks (Fig. 2). In this way edge effects are inherently disregarded.

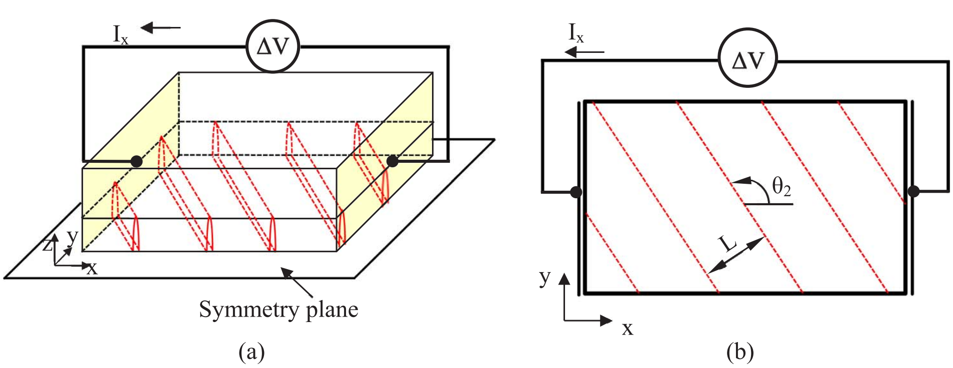

Fig. 1. (a) Schematic representation of the cracked laminate, the current is injected through the yellow areas. (b) top view of the laminate.