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Theoretical analysis on full torsional behavior of RC beams strengthened with FRP materials

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ABSTRACT

An analytical model is developed to predict the full torsional behavior of reinforced concrete (RC) beams externally wrapped with fiber reinforced polymer (FRP) materials. Based on previous modified softened truss model for torsion (MSTMT), the proposed model additionally considers the effect of concrete in tension to make a better prediction for the full torsional behavior. Reasonable compressive and tensile constitutive relationships of FRP strengthened concrete under torsion are employed in the proposed model, MSTMT-FRP-tension. Then an efficient solution algorithm is developed to accomplish the theoretical analysis. To verify the newly proposed model, a comparative analysis is conducted between the theoretical predicted torque-twist curves and the reported experimental ones of solid rectangular section strengthened with carbon fiber reinforced polymer (CFRP) or glass fiber reinforced polymer (GFRP) materials. The good agreement between the comparative results indicates the validity of the analytical model for predicting the torsional behavior of RC beams strengthened with FRP materials both at the pre-cracking and post-cracking stage.

1. Introduction

Reinforced concrete (RC) members in bridges, buildings, and other engineering structures need to be strengthened, repaired and retrofitted due to aging, environmentally induced deterioration, and upgrading of current design requirements. The fiber reinforced polymer (FRP) has been proved to be a widely used strengthening material for deficient RC members. It has various well-known advantages such as high strength to weight ratio, high corrosive resistance, and easy-to-apply character. Many significant experimental and theoretical studies in the past have been carried out to understand the flexural [1,2] and shear [3,4] behaviors of RC members externally strengthened with FRP materials since the bending moments and shear forces are regarded as primary effects, whereas the torsional strengthening has not been studied in much depth, which was just initiated in 2001 [5–7]. Torsion can be considered as primary effect, however, in some special situations such as spandrel or curved beams, eccentrically loaded bridge girders, and bridge columns under seismic load. In this case, it is important to conduct deep researches on the torsional behavior of RC members strengthened with FRP materials, including experimental, numerical and analytical investigations.

Most of the test specimens in previous experimental investigations were solid rectangular RC beams externally strengthened with carbon or glass FRP (CFRP or GFRP) materials under monotonic torsion [8–11]. Few tests of RC box beams strengthened with CFRP sheets have been conducted under monotonic torsion [12] and under cyclic torque [13]. To understand the influence of strengthening schemes of FRP system on the effectiveness of upgrading in torsional resistance of RC members, the various FRP wrapping configurations have been investigated by considering the fiber orientation, the number of beam faces strengthened, the effect of number of FRP plies used, and the influence of anchors in U-wrapped test beams [19,20]. The results have showed that the 45° spiral wrap is a much more efficient torsional strengthening scheme than vertical strips. Few researches on non-rectangular beams have been carried out in recent years. RC T-beams strengthened with different strengthening techniques under pure torsion [14] and combined shear and torsion [15] have been investigated. Spandrel RC beams strengthened with CFRP laminates also have been tested under torsion [16]. In addition, the torsional repair of damaged rectangular [17] and circular [18] RC columns with FRP materials, which helps to enhance the ultimate rotational strength, has been carried out in recent years.

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Owing to the fact that experimental investigation costs much time and money, the finite element analysis (FEA) using commercial software is a beneficial supplement to the study of torsional behavior. Numerical studies on the cracking and crushing patterns [9], the damage simulation [22], the effect of CFRP and reinforcing steel bars on the contribution to the torsional behavior [7], and the torque–twist curves [21] of RC beams were performed through FEA softwares, such as ANSYS, ABAQUS, DIANA, Algor SAP and so on.

Recent growing experimental and numerical investigations on the torsional behavior of RC members strengthened with FRP materials provide more and more valuable data to validate the rigorous analytical models for predicting the torsional behavior. The FIB [23] has provided the design equations for computing the contribution of the FRP to the torsion capacity of the RC beam strengthened with full wrapping or U-wrapping. Based on the FIB method, a simple analysis method [24] has been presented, which could compute the total contribution of steel bars and FRP wraps to the torsion capacity of the RC beam. Several analytical models based on diagonal compression field theory (CFT) have been proposed to predict the ultimate torsional strength of solid [26,27] and hollow [25] RC beams strengthened with FRP materials. Some of these models have considered the effects of various parameters including various strengthening schemes, FRP contribution, and different failure modes. What's more, the classical rotating angle softened truss model (RA-STM) for the RC members under torsion has been extended and modified in order to consider the contribution of FRP materials to the torsional capacity of the RC members [28]. Afterward, an analytical approach was proposed based on two different models: a smeared crack model for plain concrete under torsion employed in the precracking stage and a modified softened truss model for torsion (MSTMT) employed in the postcracking stage [29]. This MSTMT which considered the effect of FRP confinement on the compressive stress-strain relationship of concrete was then extended to predict the entire torsional behavior of single-cell and multicell box girders strengthened with CFRP sheets [30]. Furthermore, a new computational procedure on the basis of softened membrane model for torsion (SMMT) has been developed to predict the entire torque-twist curves of solid and hollow RC beams strengthened with FRP materials [31]. The SMMT has also been extended to consider the effect of FRP materials on the compressive stress-strain relationship of concrete, and to include a new tension stiffening relationship of concrete [22].

From above research review, it can be obtained that the existing analytical approaches for predicting the torsional behavior of RC beams strengthened with FRP materials were mostly based on either CFT, RA-STM or SMMT. Both modified CFT and RA-STM for RC members strengthened with FRP materials under torsion assume that the concrete carries no tension either before or after cracking. This assumption will result in reduction of torsional stiffness before cracking and underestimation of peak torque after cracking. Moreover, the modified SMMT for RC members strengthened with FRP materials has not considered the influence of FRP confinement on the tensile stress-strain relationship. Meanwhile, the computing procedure of algorithm for modified SMMT is quite complicated and takes much time. Therefore, to efficiently and accurately predict the entire torsional behavior of RC members strengthened with FRP materials under torsion, it is necessary to develop a new analytical model for them.

The objective of this study is to propose an effective analytical model for predicting the full torsional behavior of RC beams strengthened with FRP Materials based on the modification of the RA-STM for torsion. The proposed model considers the influence of the tensile stress in concrete on torsional behavior. Meanwhile, the effects of FRP confinement on the concrete compressive and tensile stress-strain relationships are taken into account in the analytical model. 16 specimens strengthened with CFRP or GFRP materials are selected from the available literatures to check the validation of the proposed analytical model. The specimens are strengthened with CFRP or GFRP materials in various strengthening schemes. The extensive comparisons between analytical results and experimental data are performed and discussed herein.

2. Analytical model

The analytical model adopted in this study is the Modified Softened Truss Model for Torsion with FRP considering the tension of concrete (MSTMT-FRP-tension), which is based on the analytical procedure presented by Chalioris [29], and Chai et al. [30]. When a RC beam strengthened with FRP materials is subjected to torsion, the concrete of beam carries the most of torque before cracking. Whereas, after cracking the torsion is then resisted by the space truss system composed of diagonal concrete struts, longitudinal steel bars, transverse stirrups, and FRP strengthening materials. Hence, the concrete tension should not be neglected, especially in the precracking stage. In the newly proposed model, the terms of the tensile stress σ_r are included in the in-plane equilibriums. Meanwhile, the effects of FRP confinement on the compressive and tensile stress-strain relationships of concrete are properly taken into account on the basis of compressive constitutive relationship presented by Wang [33] and tensile constitutive relationship suggested by Yang et al. [35], respectively. In addition, a trial and error algorithm for the analytical model is developed in this study to obtain the entire torque-twist curve.

2.1. Equilibrium equations

2.1.1. Bredt's equilibrium equation

The Fig. 1(a) shows a FRP strengthened rectangular RC beam subjected to an external torque *T*, which is resisted by an internal torque formed by the circulatory shear flow *q* along the periphery of the cross-section. This shear flow *q* occupies a shear flow zone, with an effective thickness t_d. Since the beam is under pure torsion, the element A in the shear flow zone is subjected to a pure shear stress $\tau_{lt} = q/t_d$ (Fig. 1(b)). Therefore, the Bredt's thin-tube theory [32] can be employed to get Eq. (1).

$$T = 2A_0 q = 2A_0 \tau_{lt} t_d \tag{1}$$

where A_0 is the area bounded by the centerline of shear flow around a cross-section.

2.1.2. In-plane equilibrium equations

After concrete cracks, as is shown in Fig. 1(b), the diagonal concrete struts are formed, inclined at an angle α to the longitudinal axis of the beam. The in-plane equilibrium equations of shear element A can be obtained by taking the wedge-shaped element (I) or (II) in Fig. 1(b) as a free body. For instance, the element (I), shown in Fig. 1(d), has the equilibriums of the *t*-direction and *l*-direction force components (*t*-direction force components: $\sum F_t = 0$, and *l*-direction force components: $\sum F_t = 0$), and then the in-plane equilibrium Eq. (2) and Eq. (3) can be derived respectively. Similarly, the element (II), shown in Fig. 1(e), has the equilibriums of the *t*-force components, and then the in-plane equilibrium Eq. (4) can be obtained. It is worth mentioning that the shear stresses in the cracks of concrete are neglected in the stress analysis of element (I) and (II) according to the principle of RA-STM.

$$\tau_{lt} = (\sigma_r + \sigma_d)\sin\alpha\cos\alpha \tag{2}$$

$$\sigma_r \sin^2\alpha - \sigma_d \cos^2\alpha + \rho_l f_l + \rho_{fl} f_{fl} = 0 \tag{3}$$

$$\sigma_r \cos^2\alpha - \sigma_d \sin^2\alpha + \rho_t f_t + \rho_{ft} f_{ft} = 0 \tag{4}$$

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