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Influences of surface and interface energies on the nonlinear vibration of laminated nanoscale plates

K.F. Wang [a], B.L. Wang [a,*], M.H. Xu [b], A.B. Yu [b]

[a] Graduate School at Shenzhen, Harbin Institute of Technology, Harbin 150001, PR China
[b] School of Materials Science & Engineering, University of New South Wales, Sydney 2052, New South Wales, Australia

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ABSTRACT

This paper studies the vibration behaviours of laminated plates with consideration of the influences of surface and interface energies. Geometric nonlinearity is taken into account in this model to obtain the results of large amplitude vibrations. Approximate closed-form solutions for simply supported plates, clamped plates and clamped circular plates are provided. Numerical results show that the surface/interface effect can affect the dynamic behaviours of laminated plates at nanometer scale. This is especially for nonlinear (large-amplitude) vibration. In addition, the ratio of the thickness to length of the plate, the external load and number of layers also affect the surface/interface effects for large amplitude vibration. This study is helpful for designing and examining the non-linear dynamic behaviour of laminated nanoplates and nanoscale devices.

1. Introduction

Laminated nanoplates and nanofilms have wide applications in different types of systems and devices, such as medical science, astronavigation and electrical industry. The research of dynamic behaviours of laminated nanoplates is necessary and helpful on the design and development of nonoscale medical equipments, detecting devices, electronic products and so on. Since the large surface to volume ratio of nanoplates, the influences of size-dependent effect caused by surface and interface energies on their mechanical properties are so significant that cannot be neglected. Accordingly, the classical mechanics formulations for laminated plates need to be modified and the results need to be recalculated. As an important mechanical property, vibration problems of nanoplates taking into account the surface energy are always one of the major research fields. Based on the fundamental works of surface effect finished by Gurtin [1] and Cammarata [2–4], many researchers made further contributions to the vibration problem of nanostructures. For example, Wang and Feng [5,6] assumed that the surfaces of the nanoscale structures as zero thickness layer with certain mechanical properties such as surface stiffness and surface residual stress. They solved some linear size-dependent problems of nanoscale beams and nanowires, such as buckling and vibration. Eremeyev et al. [7] investigated the influences of surface tension on the effective stiffness of nanoplates. By incorporating the Gurtin-Murdoch continuum elasticity into the classical plate theory, Ansari et al. [8–10] obtained the closed-form analytical solutions for the free vibration of nanoplates and indicated that the influence of surface effect relies on the sign and magnitude of the surface elastic constants. In addition to the free vibration, the forced vibration of size-dependent nanoplates was also studied. For example, Assadi [11] studied the forced vibration of a rectangular nanoplate using a generalized form of Kirchhoff plate model. Gheshlaghi and Hasheminejad [12] investigated the axisymmetric vibration of a circular nanoplate with clamped edges and obtained the lowest three natural frequencies as a function of the radius of the plates.

Nanoplates in either industrial applications or laboratorial experiments are usually very thin. In other words, most of the nanoplates have large flakiness ratios. Hence, the geometry nonlinearity should be taken into account in vibration problems. Malekzadeh et al. [13] modelled the free surfaces of the plates as two-dimensional membranes adhering to the underlying bulk material without slipping. Based on classical plate theory in conjunction with nonlocal and surface elasticity theories, they studied the nonlinear frequency parameters of the skew nanoplates. Sahmani et al. [14] developed a size-dependent higher-order shear deformable circular plate model which considers the surface elasticity, surface stress and surface density. Their research indicate that for a specified value of axial load in post buckling domain, increasing the plate thickness may leads to higher vibration frequencies. Wang and Wang [15] studied the surface effects on the nonlinear free

* Corresponding author.
E-mail address: wangbl2001@hotmail.com (B.L. Wang).

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vibration of nanoplates by using Hamilton's principle and found out the relationship between the normalized period and the thickness of nanoplates. They also proposed a finite element model to tackle the problems of bending and vibration problems of nanoplates [16].

Despite the insufficient and unsystematic research on nonlinear vibrations of laminated nanoplates, the influences of interface energy also needs to be well studied. According to the theory of Cammarata et al. [2–4], the interface energies (between different layers) and the surface energies (between the bulk and air) have significant effects on the mechanical properties of nanoplates. Early in 1998, researchers have already tried to find out the influences of interface energies in different ways. Nix and Gao [17] used a microscopic model to show that the interface stresses do work and affect the elastic straining of the interface. They also pointed out that the interface stresses in the Ag/Ni multilayered systems must be very large so that the system can be in physical equilibrium. The conclusion of interface stress obtained by Nix and Gao [17] is quite similar to the expression of surface stress demonstrated by Miller and Shenoy [18] in 2000. Chen et al. [19] focused their research on theoretical analysis and derived the generalized Young-Laplace equation of curved interfaces in nanoscale solids. Recently, Asemi et al. [20] developed a nonlocal continuum plate model for the transverse vibration of double- piezoelectric-nanoplate systems taking into account the initial stress under an external electric voltage and solved the relevant governing equations by differential quadrature method. All these research and conclusions about interface effect mentioned above demonstrate significant influences of interface energies on the mechanical properties and behaviours of multilayer nanoplates.

Due to the widely application of the multilayered nanoplates and composite nanomaterials in modern industry and technology, the associate subjects become more and more necessary. However, the current studies on nonlinear vibration problems have been focused on the single layer nanoplates. The achievements on nonlinear vibrations of laminated nanoplates are neither systematic nor sufficient, especially on forced vibrations and analytical solutions. Taking both surface/interface elasticity and surface/interface residual stress into account, this paper will derive closed-form analytical formulations and solutions of the nonlinear free and forced vibration problems of laminated nanoplates with different boundary conditions, such as simply supported rectangular plates (SSSS), clamped rectangular plates (CCCC) and clamped circular plates. Based on the theoretical developments, some case studies are provided, together with the discussions and conclusions.

2. Formulations of the model

The constitutive relationship of surface can be expressed as the following form [18]:

$$\sigma_{ij}^s = \gamma\delta_{ij} + \frac{\partial\gamma}{\partial\varepsilon_{ij}^s} \quad (1)$$

where σ_{ij}^s stands for surface stress tensor, γ stands for surface energy density and ε_{ij}^s stands for the surface strain tensor, respectively. In one-dimensional case such as nanobeams, the constitutive relations reduce to [21]:

$$\sigma^s = \tau^0 + E^s\varepsilon^s \quad (2)$$

where E^s denotes stiffness and τ^0 is the residual surface stress of the unstrained nanoplate, and can be read as [6]:

$$\tau^0 = \gamma + \frac{\partial\gamma}{\partial\varepsilon^s} \text{ at } \varepsilon^s = 0 \quad (3)$$

According to Eq. (2), the effect of surface energy can be separated into two parts: surface residual stress and surface elasticity. Wang and Feng [5,6] treated the surface as an isotropic elastic lamella with constant stiffness but without thickness and worked out the governing equations of size-dependent nanobeams by mechanical equilibrium method. As a validation of their models, Wang [22] obtained the same governing equations by variational method.

The problem under consideration is a laminated nanoplate illustrated in Fig. 1(a). The cross section of the laminated nanoplate can be seen as a combination of N layers of bulk and $N + 1$ layers of surface/interface. The thicknesses of each bulk layer are presented as h_k and the total thickness of the plate is h. For this laminated nanoplate, the equivalent Young's modulus E^* can be expressed as:

$$E^* = \frac{1}{h}\left(\sum_{k=1}^{N} E^{(k)}h_k + \sum_{k=1}^{N+1} E^{s(k)}\right) \quad (4)$$

where h_k is the thickness of the k-th layer, $E^{(k)}$ is the Young's modulus of the k-th layer, and $E^{s(k)}$ is the Young's modulus of the k-th layer surface or interface. As shown in Fig. 1, the distances from the neutral axis to each surfaces/interfaces are denoted as z_k, which can be expressed as

$$z_k = d - \sum_{n=k}^{N} h_n \quad \text{for}(k = 1, \ldots, N) \quad \text{and} \quad z_{k+1} = d \quad (5)$$

where d is the distance from the neutral axis to the bottom of the plate, and can be determined approximately from

$$\sum_{k=1}^{N+1} E^{s(k)} z_k + \frac{1}{2}\sum_{k=1}^{N} \frac{E^{(k)}}{1-\nu_k^2}\left(z_{k+1}^2 - z_k^2\right) = 0 \quad (6)$$

where ν_k is the Poisson's ratio of the kth layer. We only consider the properties of isotropic materials in this paper. Similarly to the flexural rigidity of classical laminated plates [23], the equivalent bending stiffness of laminated nanoplates can be given as

$$D^* = \sum_{k=1}^{N+1} E^{s(k)} z_k^2 + \frac{1}{3}\sum_{k=1}^{N} \frac{E^{(k)}}{1-\nu_k^2}\left(z_{k+1}^3 - z_k^3\right) \quad (7)$$

3. Nonlinear vibration of laminated nanoscale plates

3.1. Rectangular plate

There are two main methods to obtain the governing equations of nanoscale structures. One is the equilibrium of forces used by Fu et al. [24], Wang and Feng [5,6] and He and Lilley [25], and the other is the variational method derived by the energy conservation, which was used by Wang and Wang [15] and Wang [22]. In most applications, the thickness of a plate is small in comparison with its smallest dimension, and hence Kirchhoff's hypothesis may be assumed to be valid. By this assumption, the tractions on plate surface parallel to the reference plane of the plate are negligibly small as compared with the inplane stress. Accordingly, the inplane displacements are linear function of z. Therefore, the displacement of the neutral plane of the plate at a certain point x and y at time t are represented as:

$$u = u_0(x,y,t) - z\frac{\partial W}{\partial x}, \quad v = v_0(x,y,t) - z\frac{\partial W}{\partial y} \quad \text{and} \quad W = W(x,y,t) \quad (8)$$

where u and v are displacements of the neutral plane along the x and y directions, respectively and W is the deflection of the plate. Assuming there is no slipping between each layer in the model, the non-zero strain in Von-Kármán type read

$$\begin{aligned} \varepsilon_x &= \tfrac{\partial u_0}{\partial x} + \tfrac{1}{2}\left(\tfrac{\partial W}{\partial x}\right)^2 + z\kappa_x, \\ \varepsilon_y &= \tfrac{\partial v_0}{\partial y} + \tfrac{1}{2}\left(\tfrac{\partial W}{\partial y}\right)^2 + z\kappa_y, \\ \gamma_{xy} &= \tfrac{\partial u_0}{\partial y} + \tfrac{\partial v_0}{\partial x} + \tfrac{\partial W}{\partial x}\tfrac{\partial W}{\partial y} + z\kappa_{xy}. \end{aligned} \quad (9)$$