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Elastic deformation of fiber-reinforced multi-layered composite conical shell of variable stiffness

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ABSTRACT

Tailoring composite materials by fibers of spatially varying orientation angles has been realized with the advent of automated tow-placing machine. In order to use these variable stiffness composite materials as structural components, their responses to multiple external loads should be investigated. In this study, a truncated composite conical shell structure subjected to inflating pressure and surface shear traction force is considered. A mathematical model that predicts strain and stress distributions on the conical shell is developed. Based on the model, numerical examples are given for various fiber paths defined on the truncated conical shell that is subjected to inflating pressure and spatially varying shear traction force on the inner and outer surfaces, respectively. Numerical examples show that, under these external loads, the meridional strain and stress components are very sensitive to the type of fiber path definitions and value of semi-vertex angle of the cone. Boundary conditions have, also, shown remarkable effects on strain and stress distributions. To verify the adequacy of the mathematical model, the truncated composite conical shell of variable stiffness is simulated using finite element based ABAQUS commercial software.

The numerical results obtained through the developed mathematical model and ABAQUS simulations show good agreement.

1. Introduction

The use of reinforced composite materials has been advancing ever since it was introduced.

Light weight, high stiffness, toughness and easily tailorable characteristics of composite materials are the primary advantages which attract various sectors, in particular Aerospace and Automobile Industries. Aerospace applications of reinforced composite shells include aircraft's primary and secondary components, rocket propellant tanks and solid rocket motor casings for launch vehicles and missiles. Application to automotive industry incorporates high pressure fuel storage tanks for hydrogen powered automobiles.

In order to improve the structural performance of composite materials, their reinforcing fibers are tailored accordingly using automated fiber placing machine. Stress and deformation behaviors of the structure considerably depend on the orientation of these reinforcing fibers. Due to anisotropic nature of composite materials, predicting spatial variation of stress and deformation is challenging. In particular, when the reinforcing fibers are tailored in such a way that their orientation angles vary spatially, the case is even more complicated. In a conventional composite laminate, each ply is reinforced by fibers of constant orientation angle. The required performance of composite structure can be obtained by designing ply-stacking sequence and orientation of reinforcing fibers in each ply. The reinforcing fibers are limited to constant orientation angles throughout the laminate. Such conventional laminate has high stiffness in the fiber direction and low stiffness in the transverse to the fiber direction throughout its dimensions.

To meet a required structural performance, spatial variation of stiffness of composite structure may become indispensable. A composite structure with spatially varying stiffness is termed as variable stiffness composite. This spatial dependent stiffness of a given composite structure can be achieved by spatially varying fiber density [1–7], varying laminate thickness (by adding or dropping plies at different locations) [8–11], varying material composition and properties (functionally graded materials) [12], or varying fiber orientation angles [13–17]. As a part of advancement in composite manufacturing technology, reinforcing fibers are being tailored in curvilinear manner. This opens a wider

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opportunity to manufacture composite structures with better performance that takes local effects into consideration. The automated fiber placing machine places the fiber tow following a per-determined path that can be used as a reference path. Once the reference path is defined, consecutive tow placing can be done either by shifting the path parallel to the reference path or perpendicular to the direction of fiber orientation variation. The perpendicular tow shifting method, however, introduces defects like fiber overlap and kinks. A recent study by Kim et al. [18] shows a continuous tow shearing method can overcome these drawbacks.

Most research works are focused on enhancing buckling load carrying capacity [19–25], vibration analysis [26–32], bending stiffness [33], and optimizing structural weight. Though variable stiffness composite structures draw an increasing attention, analytical formulations that predict their responses to multiple external loads remains rare. To the knowledge of the authors there is no previous work on strain and stress analysis for composite truncated conical shell structures subjected to spatially varying external loads. This work deals with the formulation of mathematical model that predicts strain and stress distributions on elastically deformed composite truncated conical shells of variable stiffness subjected to inflating pressure and spatially varying surface traction force. The stiffness variability of this conical shell is due to the variation of reinforcing fiber orientation along the meridional direction. Truncated conical shell structures are very common in aeronautical and Aerospace Engineering, Nuclear power plants, Ship and Submarine hulls. There are possibilities that components of these structures maybe replaced by composite shells of variable stiffness. External loads such as inflating pressure and surface traction forces (drag forces) are common in some of these structures. Hence, the ultimate objective of this study is to investigate stress distribution and deformation behavior of truncated conical shell subjected to these loads.

The subsequent sections, herein, are organized as follows. In Section Two, a mathematical model that predicts elastic deformation and stress distribution in the truncated composite conical shell is developed. In Section Three, the approaches taken to implement a Fortran subroutine into finite element method based ABAQUS commercial software that is used to model the composite truncated conical shells of variable stiffness are discussed. Based on the illustrative numerical examples, results and discussions are provided in Section Four and conclusions are given in Section Five.

2. Mathematical model

In this mathematical formulation, a truncated conical shell of semi-vertex angle (α), smaller radius R_1, and larger radius R_2, shown in Fig. 1a, is considered. The deformation of this truncated conical shell is considered under two cases. For the first case, the cone is fixed at it's truncated edge (with smaller radius R_1) while the other edge is free. For the second case, both edges of the cone are freed.Let the initial location of a material particle on the conical shell be defined in a convective orthogonal curvilinear coordinate (ξ, ϑ, η). Here, ξ and ϑ are the meridional and circumferential axes, respectively, whereas η is an axis normal to the lateral surface of the conical shell and points outward. The origins of Cartesian coordinate system X, Y, Z and cylindrical coordinate system R, ϑ, Z coincide at the radial center of the truncated edge of the cone.

As shown in Fig. 1a, the initial location of a given material particle with respect to the bases vectors $\hat{i}, \hat{j}$, and $\hat{k}$ of the coordinate axes X, Y, and Z, respectively, is defined by the position vector ρ_o as:

$$\rho_o(Z, \vartheta, R) = R\left\{\cos(\Theta)\hat{i} + \sin(\Theta)\hat{j}\right\} + Z\hat{k} \tag{2.1}$$

where

$$\begin{aligned} Z &= \{\xi - \eta\tan(\alpha)\}\cos(\alpha) = \xi\cos(\alpha) - \eta\sin(\alpha) \\ R &= Z\tan(\beta + \alpha) \end{aligned} \tag{2.2}$$

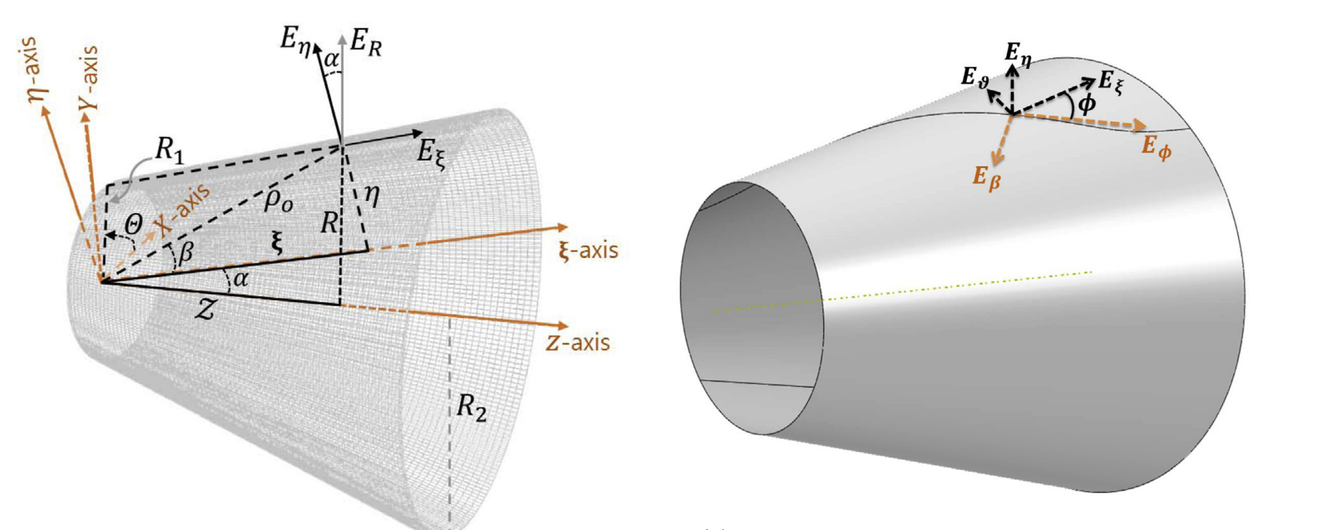
Expanding $\tan(\beta + \alpha)$ and using the relation $\tan(\beta) = \eta/\xi$, the follow expression can be obtained.

$$R = \eta\cos(\alpha) + \xi\sin(\alpha) \tag{2.3}$$

Substituting Eqs. (2.2) and (2.3) into Eq. (2.1), the position vector ρ_o can be re-written as:

$$\begin{aligned} \rho_o(\xi, \vartheta, \eta) = &\{\eta\cos(\alpha) + \xi\sin(\alpha)\}\{\cos(\Theta)\hat{i} + \sin(\Theta)\hat{j}\} \\ &+ \{\xi\cos(\alpha) - \eta\sin(\alpha)\}\hat{k} \end{aligned} \tag{2.4}$$

The bases vectors E_ξ, E_ϑ, and E_η along coordinate axes ξ, ϑ, and η, respectively, are derived with respect to Cartesian coordinate bases $(\hat{i}, \hat{j}, \hat{k})$ as:



(a) Coordinate systems on truncated conical shell

(b) Variable fiber orientation on the truncated conical shell

Fig. 1. Coordinate systems and variable fiber path on truncated conical shell.