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Semi-analytical solutions for static analysis of piezoelectric laminates

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ABSTRACT

Displacement and stress analysis of an all-round simply supported piezoelectric laminate has been carried out with a new model. Formulation part has been developed using elasticity approach and the methodology is free from any a-priory assumption in thickness direction. The mathematical model is a set of mixed first ordered ordinary differential equations (ODEs). Solution is obtained using numerical integration in through-thickness direction. Accuracy of the proposed model is assessed by comparing numerical results for single layer piezoelectric plate as well as for multi-layered laminates with exact solutions available in literature and are found to be in good agreement with the same. Additional investigation has been performed to establish a few future references.

1. Introduction

Piezoelectric material develops electric charge when mechanically strained, an effect called as direct piezoelectricity. As an inverse effect, it undergoes mechanical deformation when subjected to electric potential [1].This reversible electro-mechanical coupling is judiciously utilized in a smart or intelligent material. A smart material comprises of layers of elastic substrate with patches of piezoelectric sensors and actuators. Behavior of such a material becomes self-governing, when coupled with a circuit strategy, and controls deflections due to static loads and vibrations due to dynamic loads.

Piezoelectricity was discovered by Pierre Curie and Jacques Curie in 1880 [1]. However, for about a century, it remained to be just a natural wonder. Exhaustive research on piezo-materials as regards engineering applications began in the decade of 1980. Since then, researchers have contributed immensely towards the analysis of smart materials. Tiersten and Mindlin [2] initiated the work in piezoelectric plates. Tiersten [3] proposed the governing constitutive relation of piezoelectricity. Ray et al. [4,5] developed elasticity solutions for polyvinyledene difluoride (PVDF) plate under cylindrical bending and for smart laminates under bi-directional bending. Heyliger [6,7] developed exact solutions for composite laminate attached with layers of lead zirconate titanate (PZT) and for a piezoelectric bimorph. Vel and Batra [8] used Eshelby–Stroh formulation to obtain 3D elasticity solution to analyze multilayered piezoelectric plate for arbitrary boundary conditions (BCs). Pendhari et al. [9] provided semi-analytical solutions for cylindrical bending of simply supported single layer piezoelectric plate. Alibeigloo and Madolia [10] used differential quadrature method and Fourier series approach for static analysis of cross-ply laminated piezo-plates.

Exact solutions obtained by solving governing partial differenetial equations (PDEs) are important because they represent near-accurate response of the material. However, these are achievable only for simple geometry, loading and constrains. For arbitrary conditions, researchers have proposed models using approximate theories. Based on Classical plate theory (CPT), Dimitridis et al. [11], Crawley and Lazarus [12], Wang and Rogers [13], Jayakumar et al. [14] analyzed piezoelectric plates. Lee and Moon [15] and Lee [16] presented consistent piezoelectric plate theory based on assumptions of CPT. In the models proposed using shear deformation theories, the notable contributions, amongst many, came from Chandrashekhara and Agarwal [17], Jonnalagadda et al. [18], Huang and Wu [19], Wu et al. [20], Liao and Yu [21] by First order shear deformation theory (FOST) and from Ray et al. [22], Kim et al. [23], Amadeu et al. [24], Beheshti-Aval et al. [25] by Higher order shear deformation theory (HOST). Carrera [26] developed improved Reissner–Mindlin type model for analysis of smart laminates. Ballhause et al. [27] and Carrera et al. [28] proposed unified formulation for the electromechanical analysis of smart plates. Gibson [29] presented an exhaustive review on the recent research developments in multifunctional composites.

In the present work, semi-analytical model proposed by Kant et al. [30] is developed for the analysis of piezoelectric laminates.

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Notations

a, b, h length, width and thickness of plate	$\varepsilon_x, \varepsilon_y, \varepsilon_z$ normal strains in x, y, z directions
E_1, E_2, E_3 elastic moduli in principal directions	$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ shear strains in xy, xz, yz planes
$\nu_{ij}, i, j = 1, 2, 3$ generalized Poisson's ratios	E_x, E_y, E_z electric field intensities in x, y, z directions
u, v, w displacements in x, y, z directions	$e_{ij}, i, j = 1, 2, \ldots, 6$ piezoelectric constants
$\sigma_x, \sigma_y, \sigma_z$ normal stresses in x, y, z directions	D_x, D_y, D_z electric displacements in x, y, z directions
$\tau_{xy}, \tau_{xz}, \tau_{yz}$ shear stresses in xy, xz, yz planes	B_x, B_y, B_z body force intensities in x, y, z directions

Displacements, transverse normal stress, transverse shear stresses, electric potential and transverse electric displacement have been considered as primary variables. A simply supported smart laminate subjected to electro-mechanical loading is formulated as a mixed two-point boundary value problem (BVP) in the interval $-h/2 \leqslant z \leqslant h/2$ with half of the variables specified at the edges $z = \pm h/2$.

2. Mathematical formulation

A smart laminate consisting of substrate of isotropic/orthotropic material layers with a piezoelectric layer at top and bottom face each, acting respectively as actuator and sensor is considered (Fig. 1). An all-round simple diaphragm support (SSSS) is assumed along the longitudinal edges, $x = 0, a$ and $y = 0, b$. Longitudinal edges of the laminate are grounded with zero potential. The laminate is subjected to transverse mechanical and electrical loading at the top surface.

Coupled elastic and electric field equations due to Tirsten [3], 3D elasticity equilibrium equations, 3D strain–displacement equations and 3D charge equilibrium equation due to Maxwell [31] are respectively;

$$\{\sigma\} = \left[C^E\right]\{\varepsilon\} - [e]\{E\}, \quad \{D\} = [e]^T\varepsilon + \left[\varepsilon^S\right]\{E\} \tag{1}$$

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} + B_x = 0, \quad \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + B_y = 0,$$
$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\sigma_z}{\partial z} + B_z = 0 \tag{2}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \tag{3}$$

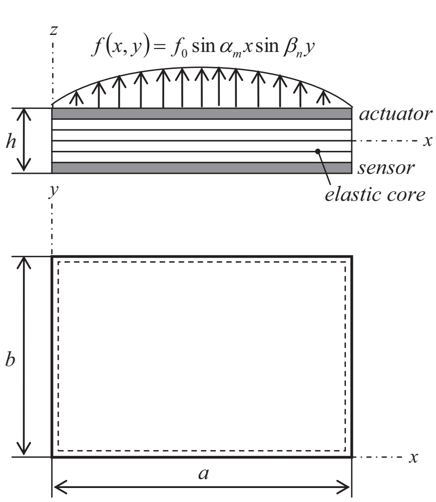

Fig. 1. Smart laminate configuration.

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 \tag{4}$$

where $[C^E]$ is elasticity matrix at constant electric field and $[\varepsilon^S]$ is dielectric constant matrix at constant strain.

Eq. (1) may be expanded as;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} -\partial\phi/\partial x \\ -\partial\phi/\partial y \\ -\partial\phi/\partial z \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} -\partial\phi/\partial x \\ -\partial\phi/\partial y \\ -\partial\phi/\partial z \end{Bmatrix} \tag{6}$$

in which,

$$C_{11} = \frac{E_1(1 - \nu_{23}\nu_{32})}{\Delta}; \quad C_{12} = \frac{E_1(\nu_{21} + \nu_{31}\nu_{23})}{\Delta};$$
$$C_{13} = \frac{E_1(\nu_{31} + \nu_{21}\nu_{32})}{\Delta}; \quad C_{22} = \frac{E_2(1 - \nu_{13}\nu_{31})}{\Delta};$$
$$C_{23} = \frac{E_2(\nu_{32} + \nu_{12}\nu_{31})}{\Delta}; \quad C_{33} = \frac{E_3(1 - \nu_{12}\nu_{21})}{\Delta};$$
$$C_{44} = G_{12}; \quad C_{55} = G_{13}; \quad C_{66} = G_{23} \quad \text{where}$$
$$\Delta = (1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{12}\nu_{23}\nu_{31}) \tag{7}$$

The piezoelectric constant matrix $[e]$ in Eqs. (5), (6) and dielectric constant matrix $[g]$ in Eq. (6) are due to Cady [1] and Tzau and Pandita [32] respectively.

Eqs. (1)–(4) have a total of 19 unknowns; $u, v, w, \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}, \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}, D_x, D_y, D_z$ and ϕ in 19 equations. However, these 19 unknowns are not entirely independent. After some algebraic manipulation of the above sets of equations, a set of PDEs involving only eight primary variables; $u, v, w, \sigma_z, \tau_{yz}, \tau_{xz}, D_z$ and ϕ is obtained as;