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External boundary effects on the velocity profile for generalized Newtonian fluid flow inside a homogeneous porous medium

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ABSTRACT

Analytical expressions for the velocity profiles of different types of generalized Newtonian fluids are derived for fluids traversing a porous domain enclosed between two stationary parallel plates, as well as for free-flow over and flow through a porous domain. In the first scenario a Brinkman-like equation was solved where a no-slip boundary condition was enforced at the macroscopic external boundaries. In the composite domain, analytical expressions have been derived by matching Stokes flow to the solution of the Brinkman equation after assuming a continuity in the shear stress at the fluid-porous interface. For shear thinning and shear thickening fluids, in both scenarios, an inverse approach had to be considered to obtain the average velocity profile within the porous region. The permeability of a porous structure for a specific traversing fluid subjected to a specific pressure gradient is approximated by means of a representative unit cell that is applicable to an infinite porous region. The validity of this analytical modeling approach is established by comparing it to numerical results obtained from an in house developed numerical code.

1. Introduction

In this paper analytical expressions are derived for the average velocity profiles of generalized Newtonian fluids traversing a porous medium subjected to different macroscopic exterior boundary conditions. Firstly a porous medium restricted between two stationary parallel plates is considered and secondly a large porous domain adjacent to an open channel is considered where flow occurred in and over the porous region. The study of such flow phenomena has practical applications in fields such as geophysics, mechanical and process engineering, as well as physiology. Such analytical expressions are also very useful for validating numerical simulations. Numerical solutions to the average velocity profiles are also obtained by means of an in house developed numerical code.

Numerous analytical studies have been published on Newtonian fluids traversing a composite channel (a porous domain adjacent to a free-flow channel). We shall briefly review earlier work here. In a study by Beavers and Joseph [3], Stokes flow was assumed in the free-flow channel and Darcy's law was assumed to be adhered to in the porous domain below a transition region of unknown width. The velocity profile within the transition layer was not modeled analytically. They observed from experimental results that the velocity at the fluid-porous interface, $u^{q}_{\omega\alpha}$ (see Fig. 10), is much greater than the Darcy velocity, u^{q}_{D}. The following semi-empirical ad hoc boundary condition was enforced at the $\omega\alpha$-boundary:

$$\left.\frac{du^{q}_{\alpha}}{dy}\right|_{y=0^{+}} = \frac{\alpha_{BJ}}{\sqrt{K_D}}\left(u^{q}_{\omega\alpha} - u^{q}_{D}\right), \tag{1}$$

where α_{BJ} is a fluid independent, empirical slip coefficient that characterizes the local geometry of the porous structure at the interfacial layer.

In a later study, Neale and Nader [13] completed the average velocity profile of Beavers and Joseph [3] by employing the Brinkman equation in the transition layer. They assumed a shear stress continuity at the $\omega\alpha$-interface:

$$\mu\left.\frac{du^{q}_{\alpha}}{dy}\right|_{y\to 0^{+}} = \mu_{\omega}\left.\frac{du^{q}_{\omega}}{dy}\right|_{y\to 0^{-}}. \tag{2}$$

In Eq. (2), μ_{ω} denotes the effective shear rate independent viscosity of a Newtonian fluid within a porous medium that differs from the fluid viscosity, μ. There is therefore a shear rate discontinuity at the interface.

To obtain a continuous velocity profile across the $\omega\alpha$-interfacial boundary, Ochoa-Tapia and Whitaker [14] introduced a stress-jump boundary condition by incorporating the excess stresses

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initiated by the homogenous assumption made regarding the porous structure up to the $\omega\alpha$-boundary in the volume averaging procedure. Applying a volume averaging technique, the following boundary condition was enforced at the fluid-porous interface:

$$\left.\frac{du_{\alpha}^{q}}{dy}\right|_{y\to 0^{+}}=\frac{1}{\varepsilon}\left.\frac{du_{\omega}^{q}}{dy}\right|_{y\to 0^{-}}-\frac{\beta_{\mathrm{OW}}}{\sqrt{K_{\mathrm{D}}}}u_{\omega\alpha}^{q}. \tag{3}$$

In Eq. (3), β_{OW} is the empirically adjustable stress-jump coefficient introduced by Ochoa-Tapia and Whitaker [14]. The constant porosity, ε, denotes the porosity of the homogenous ω-region of the porous domain. Comparing Eqs. (2) and (3) it therefore follows that $\mu_{\omega}=\mu/\varepsilon$. The shear rate discontinuity resulting from the condition enforced by Neale and Nader [13] at the fluid-porous interface (Eq. (2)) therefore increases as the porosity decreases.

In this study, the method of Neale and Nader [13] is followed in association with volume averaging for the modeling of purely viscous non-Newtonian fluids. In this paper the excess stresses mentioned above are neglected and homogeneity is assumed up to the fluid-porous interface.

2. Analytical background

2.1. Governing equation on continuum level and the fluid model

Utilizing the continuity equation for mass conservation, the momentum transport equation governing flow on a continuum level is given by

$$\frac{\partial}{\partial t}(\rho\underline{v})+\nabla\cdot(\rho\underline{v}\underline{v})=-\nabla p+\nabla\cdot\underline{\underline{\tau}}+\underline{F}_{\mathrm{b}}. \tag{4}$$

Here ρ denotes the density of the fluid, p the static pressure, $\underline{\underline{\tau}}$ is the second order shear stress tensor and $\underline{F}_{\mathrm{b}}$ denotes the body forces, e.g. gravitation. In this study the fluid is assumed to be incompressible and the density is assumed to be constant with respect to both time and space. Since only generalized Newtonian fluids are considered in this paper, the shear stress may be written in the following form:

$$\underline{\underline{\tau}}=\eta(|\underline{\underline{\dot{\gamma}}}|)\underline{\underline{\dot{\gamma}}}. \tag{5}$$

Here $\underline{\underline{\dot{\gamma}}}$ denotes the strain rate tensor and η is the apparent viscosity which is a function of the magnitude of the strain rate tensor that is defined as

$$|\underline{\underline{\dot{\gamma}}}|=\sqrt{\frac{1}{2}(\underline{\underline{\dot{\gamma}}}:\widetilde{\underline{\underline{\dot{\gamma}}}})}. \tag{6}$$

The tilde denotes the transpose of the second order tensor.

A Herschel–Bulkley fluid is a viscoplastic fluid that may exhibit either shear thinning or shear thickening behavior once the yield stress has been exceeded. Under simple shearing conditions, this fluid model may be expressed as follows:

$$\begin{aligned}&\dot{\gamma}=0 && \text{for } |\tau|\leqslant|\tau_y|,\\ &\tau=(|\tau_y|+K|\dot{\gamma}|^{n})\,\mathrm{sign}(\dot{\gamma}) && \text{for } |\tau|>|\tau_y|.\end{aligned} \tag{7}$$

The scalar shear stress, τ, is defined such that $\mathrm{sign}(\tau)=\mathrm{sign}(\dot{\gamma})$, where $\dot{\gamma}$ represents the shear rate. In Eq. (7), $|\tau_y|$ is the yield stress, K the consistency index and n denotes the behavior index. If the behavior index is greater than unity, the fluid is shear thickening and if it is less than unity ($n>0$), the fluid is shear thinning. The Herschel–Bulkley fluid model reduces to that of a power law fluid if $|\tau_y|=0$ and to a Bingham plastic fluid model if $n=1$. If both these conditions are applicable, this fluid model reduces to that of a Newtonian fluid where K then represents the constant viscosity, μ.

2.2. Volume averaged governing equation for flow through porous media

Standard volume averaging techniques [1] are implemented. Eq. (4) is averaged over a representative elementary volume (REV), where its volume, $\mathcal{U}_o$, consists of both fluid and solid volumes. Assuming that the porous structure is rigid, that the fluid is homogeneous and that the no-slip boundary condition is satisfied at the interstitial fluid-solid interfacial surfaces, $\mathcal{S}_{fs}$, the volume averaged momentum transport equation follows:

$$\begin{aligned}&\frac{\partial}{\partial t}[\rho(\varepsilon\widehat{\underline{u}})]+\nabla\cdot[\rho\widehat{\underline{u}}(\varepsilon\widehat{\underline{u}})]\\ &\quad=-\varepsilon\nabla\langle p\rangle_f+\nabla\cdot[\langle\eta\rangle_f\nabla(\varepsilon\widehat{\underline{u}})]+\nabla(\varepsilon\widehat{\underline{u}})\cdot\nabla\langle\eta\rangle_f\\ &\quad-\frac{1}{\mathcal{U}_o}\iint_{\mathcal{S}_{fs}}(\{p\}\underline{n}-\underline{n}\cdot\eta\nabla\underline{v})\,d\mathcal{S}+\langle\underline{F}_{\mathrm{b}}\rangle_o.\end{aligned} \tag{8}$$

In Eq. (8), $\widehat{\underline{u}}$ is the intrinsic phase average of the velocity and $\underline{v}$ denotes the velocity on a continuum level in the interstitial pores. The unit vector $\underline{n}$ is directed towards the solid phase at the fluid–solid interfaces. The intrinsic phase average and the phase average of a quantity are denoted by $\langle\rangle_f$ and $\langle\rangle_o$ respectively. The deviation at a point with respect to the intrinsic phase average is denoted by $\{\}$.

In the derivation of Eq. (8), following Bear and Bachmat [2], the momentum dispersion term was assumed to be negligible in comparison to the macroscopic convection rate (the second term on the left hand side of Eq. (8)). It is also assumed here that

$$\langle\eta\rangle_f\langle\nabla\underline{v}+\widetilde{\nabla\underline{v}}\rangle_o\gg\langle\{\eta\}\{\nabla\underline{v}+\widetilde{\nabla\underline{v}}\}\rangle_o. \tag{9}$$

Eq. (8) will be applied to flow through an infinite porous region (Section 3), fluid traversing a porous region enclosed between two parallel plates (Section 4) and flow through a composite channel consisting of an infinite porous domain located adjacent to a free-flow channel (Section 5).

3. An infinite porous region

3.1. Analytical models

With reference to Cloete and Smit [7], a representative unit cell (RUC) is used to find an estimate of the apparent permeability for different fluid types traversing an infinite porous domain or a setup where the effects due to the macroscopic boundaries are assumed to be negligible. This method was referred to as secondary averaging in the said study.

If the phase average of the velocity (in this paragraph, also the superficial velocity), $\langle\underline{v}\rangle_o$, is assumed to be time independent and uniform, since the density of the considered fluid is constant, Eq. (8) simplifies to

$$-\nabla\langle p\rangle_f+\langle\underline{F}_{\mathrm{b}}\rangle_f=\frac{1}{\mathcal{U}_f}\iint_{\mathcal{S}_{fs}}(\{p\}\underline{n}-\underline{n}\cdot\eta\nabla\underline{v})\,d\mathcal{S}, \tag{10}$$

where $\mathcal{U}_f$ denotes the volume occupied by fluid in the REV, i.e. $\mathcal{U}_f=\varepsilon\mathcal{U}_o$. From here on, for flow through a homogeneous porous medium where the effects of the external boundaries are negligible, the superficial velocity is denoted by $\widehat{\underline{q}}_{\mathrm{H}}$. In the low Reynolds number Darcy regime, Eq. (10) may be written in the following form:

$$-\nabla\langle p_{\mathrm{b}}\rangle_f=\frac{K}{K_{\mathrm{D}}}q_{\mathrm{H}}^{n-1}\widehat{\underline{q}}_{\mathrm{H}}, \tag{11}$$

where the b-subscript implies that the body forces are incorporated as part of the pressure gradient. In Eq. (11), K_{D} denotes the apparent permeability which, for non-Newtonian fluids, is not only dependent on the characteristics of the porous medium (therefore