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Buckling analysis of laminated plate structures with elastic edges using a novel semi-analytical finite strip method

Qingyuan Chen [a], Pizhong Qiao [a,b,*]

[a] State Key Laboratory of Ocean Engineering, Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] Department of Civil and Environmental Engineering, Washington State University, WA 99164-2910, USA

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ABSTRACT

A novel semi-analytical finite strip method is presented for buckling analysis of composite plate structures with boundary edges elastically supported. A set of unique Fourier series functions is introduced to represent the longitudinal variation of deflection along a strip, and they are capable of handling elastic edges with translational and rotational spring supports. The proposed hybrid method overcomes limitation of classical finite strip method only capable of handling simple end boundary conditions of structures, and it avoids the ill-conditioning when a set of standard Fourier series functions is used for buckling analysis. Accuracy and validity of the proposed method are demonstrated by the convergence and comparative studies in comparison with the numerical finite element method. As an example, the present method is applied to buckling analysis of a composite Z-stiffened panel under pure shear, and its capability and efficiency of treating different edge conditions in the panel skin and stiffeners are illustrated.

1. Introduction

The semi-analytical finite strip method has been proposed by Cheung [1,2] for several decades, and there have been a great amount of developments in the method itself and in its application [3–19]. However, this technique is fraught with limitation of only handling simple end boundary conditions of structures (i.e., both ends simply supported, both ends clamped, one end simply supported and the other end clamped, both ends free, or one end clamped and the other end free). The structures with the end either simply supported, clamped or free are extreme cases; while in reality, the end edges of structures are usually elastically supported or restrained by adjacent components. When it comes to these complicated and more realistic end boundary conditions, many scholars turn to the already developed spline finite strip method [2].

Many researchers have investigated the vibration of plates with elastically-restrained boundary condition edges using series solutions. For example, Li et al. [20] developed an analytical method for vibration analysis of rectangular isotropic plates with elastically-restrained edges, in which the displacement solution is expressed as a combination of a standard Fourier cosine series and several auxiliary closed-form functions. Beslin and Nicolas [21] proposed a hierarchical set with trigonometric functions for flexural vibration of rectangular isotropic plates. Barrette [22] used the hierarchical trigonometric functions of Beslin and Nicolas [21] as local trial functions in prediction of stiffened plate vibration, and the local functions are defined on the plate domain present between consecutive stiffeners. Then, Dozio [23–25] used this set of trigonometric functions with the Ritz method for general vibration of rectangular isotropic plates and in-plane vibration analysis of isotropic and composite plates, and all the plates considered have the arbitrary elastic boundaries. However, the set of trigonometric functions proposed by Li et al. [20] and Beslin and Nicolas [21] cannot be used for buckling analysis of isotropic and composite rectangular plates with complex and restrained boundary conditions, because of ill-conditioning. While in the authors' opinion, the incapability of considered trigonometric functions for buckling analysis is caused by many terms in the geometric matrices being zero.

In this paper, a novel semi-analytical finite strip method is presented for buckling analysis of laminated composite structures under shear and compressive loading, in which the longitudinal series functions are replaced by the set of trigonometric functions as proposed by Beslin and Nicolas [21]. The proposed novel semi-analytical finite strip method retains the advantages of the finite

* Corresponding author at: Department of Engineering Mechanics, Shanghai Jiao Tong University, Shanghai 200240, PR China.

E-mail addresses: qiao@sjtu.edu.cn, qiao@wsu.edu (P. Qiao).

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strip method (FSM), and at the same time it overcomes the shortcoming of the existing FSM only capable of handling the classical and simple end boundary condition of structures; more important, the hybrid method allows the use of the set of trigonometric functions proposed by Beslin and Nicolas [21] without ill-conditioning.

2. Theoretical formulations

Consider a typical finite strip element with the length L and the width Be (Fig. 1). k_{y0}^{T} and k_{y1}^{T} are the translational (vertical) spring stiffness coefficients along the ends of the finite strip element of $y=0$ and $y=L$, respectively; k_{y0}^{R} and k_{y1}^{R} are the rotational spring stiffness coefficients along the ends of the finite strip element of $y=0$ and $y=L$, respectively.

Based on the Classical Laminated Plate Theory (CLPT), the displacements of the middle surface of the plate $u(x,y)$, $v(x,y)$ and $w(x,y)$ are expressed by the interpolation polynomial function in x-direction and smooth series functions in y-direction:

$$\begin{aligned} u &= \sum_{m=1}^{r} Y_u^m \{C_u\}\{\delta\}_m^e \\ v &= \sum_{m=1}^{r} Y_v^m \{C_v\}\{\delta\}_m^e \\ w &= \sum_{m=1}^{r} Y_w^m \{C_w\}\{\delta\}_m^e \end{aligned} \tag{1}$$

where $\{\delta\}_m^e$ is a vector representing the mth term nodal displacement parameters at the nodal lines of the finite strip element. For the low order finite strip with three nodal lines (LO3, see Fig. 1), the following expression is held:

$$\{\delta\}_m^e = \{u_{im}\ \ v_{im}\ \ w_{im}\ \ \theta_{im}\ \ u_{km}\ \ v_{km}\ \ w_{km}\ \ \theta_{km}\ \ u_{jm}\ \ v_{jm}\ \ w_{jm}\ \ \theta_{jm}\}^{\mathrm{T}} \tag{2}$$

where θ_{im}, θ_{jm} and θ_{km} are the rotation parameters at the three longitudinal nodal lines, respectively, and they are defined as $\theta = \partial w/\partial x$. $\{C_u\}$, $\{C_v\}$, $\{C_w\}$ are the transverse interpolation shape functions, and they are given by

$$\begin{aligned} \{C_u\} &= \{C_1\ \ 0\ \ 0\ \ 0\ \ C_2\ \ 0\ \ 0\ \ 0\ \ C_3\ \ 0\ \ 0\ \ 0\} \\ \{C_v\} &= \{0\ \ C_1\ \ 0\ \ 0\ \ 0\ \ C_2\ \ 0\ \ 0\ \ 0\ \ C_3\ \ 0\ \ 0\} \\ \{C_w\} &= \{0\ \ 0\ \ C_4\ \ C_5\ \ 0\ \ 0\ \ C_6\ \ C_7\ \ 0\ \ 0\ \ C_8\ \ C_9\} \end{aligned},$$

where, $C_1 = 1-3\bar{x}+2\bar{x}^2$, $C_2 = 4\bar{x}-4\bar{x}^2$, $C_3 = -\bar{x}+2\bar{x}^2$, $C_4 = 1-23\bar{x}^2+66\bar{x}^3-68\bar{x}^4+24\bar{x}^5$, $C_5 = \bar{x}-6\bar{x}^2+13\bar{x}^3-12\bar{x}^4+4\bar{x}^5$, $C_6 = 16\bar{x}^2-32\bar{x}^3+16\bar{x}^4$, $C_7 = -8\bar{x}^2+32\bar{x}^3-40\bar{x}^4+16\bar{x}^5$, $C_8 = 7\bar{x}^2-34\bar{x}^3+52\bar{x}^4-24\bar{x}^5$, $C_9 = -\bar{x}^2+5\bar{x}^3-8\bar{x}^4+4\bar{x}^5$, $\bar{x} = x/Be$.

The longitudinal series functions Y_u^m, Y_v^m, Y_w^m are defined as:

$$Y_u^m = Y_v^m = Y_w^m = \phi_m(\eta) \tag{3}$$

where $\eta = y/L$, and the admissible series function is proposed by Beslin and Nicolas [21] as:

$$\phi_m(\eta) = \sin(a_m\eta + b_m)\sin(c_m\eta + d_m) \tag{4}$$

where the coefficients a_m, b_m, c_m and d_m are listed in Table 1, and the first eight series functions $\phi_m(\eta)$ defined by Eq. (4) are reported in Table 2. Note that the series functions when $m > 4$ have zero displacement and zero slope at the ends of the plate strip element; the free deflection and rotation at the end of $y=0$ are controlled by the first function $\phi_1(\eta)$ and the second function $\phi_2(\eta)$; and the non-zero displacement and slope at the end of $y=L$ are dominated by the third function $\phi_3(\eta)$ and the fourth function $\phi_4(\eta)$.

The plate stiffness equations are expressed as:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B]^{\mathrm{T}} \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix} \tag{5}$$

with

$$\begin{aligned} &\{N\} = \{N_x\ \ N_y\ \ N_{xy}\}^{\mathrm{T}}, \quad \{\varepsilon^0\} = \{\varepsilon_x^0\ \ \varepsilon_y^0\ \ \gamma_{xy}^0\}^{\mathrm{T}} \\ &\{M\} = \{M_x\ \ M_y\ \ M_{xy}\}^{\mathrm{T}}, \quad \{\kappa\} = \{\kappa_x\ \ \kappa_y\ \ \kappa_{xy}\}^{\mathrm{T}} \\ &[A] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \quad [B] = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}, \\ &[D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \end{aligned} \tag{6}$$

where N_x, N_y and N_{xy} are, respectively, the membrane transverse, longitudinal and in-plane shear forces per unit length, and ε_x^0, ε_y^0 and γ_{xy}^0 are the membrane strains; M_x, M_y and M_{xy} are, respectively, the transverse and longitudinal bending and twisting moments per unit length, and κ_x, κ_y and κ_{xy} are the flexural strains, known as the curvatures. A_{ij}, B_{ij} and D_{ij} are, respectively, the extensional, bending-extension, and bending stiffness coefficients.

The strain energy of the plate finite strip element can be expressed as

$$\begin{aligned} U^e &= \frac{1}{2}\iint \left\{ \{\varepsilon\}^{\mathrm{T}} \quad \{\kappa\}^{\mathrm{T}} \right\} \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix} \mathrm{d}x\mathrm{d}y \\ &= \iint \left(\frac{1}{2}\{\varepsilon\}^{\mathrm{T}}[A]\{\varepsilon\} + \kappa^T[B]\{\varepsilon\} + \frac{1}{2}\{\kappa\}^T[D]\{\kappa\} \right)\mathrm{d}x\mathrm{d}y \end{aligned} \tag{7}$$

Substituting the related variables in Eqs. (1)–(6) into Eq. (7), the strain energy equation is written as

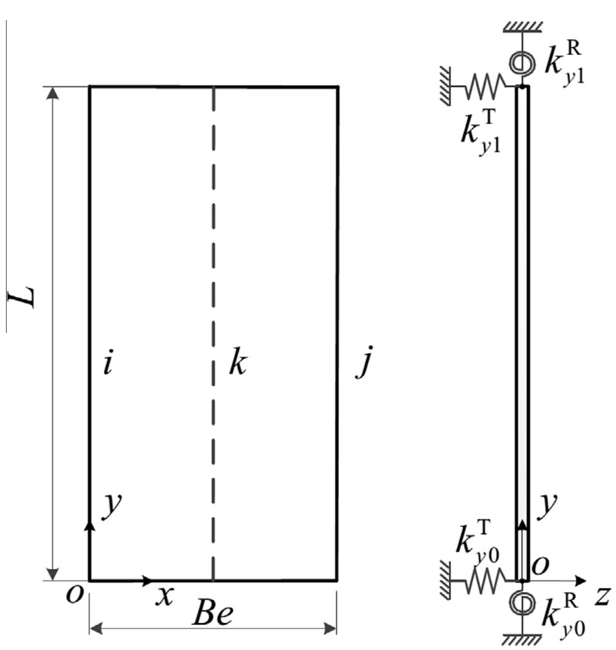

Fig. 1. LO3 plate finite strip element.

Table 1
Coefficients of the series function.

m	a_m	b_m	c_m	d_m
1	$\pi/2$	$3\pi/2$	$\pi/2$	$3\pi/2$
2	$\pi/2$	$\pi/2$	$-\pi$	$-\pi$
3	$\pi/2$	0	$\pi/2$	0
4	$\pi/2$	0	π	0
>4	$(m-4)\,\pi$	$(m-4)\,\pi$	π	π