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Heat transfer enhancement in laminar impinging flows with a non-newtonian inelastic fluid

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ABSTRACT

Non-Newtonian rheology, even when relatively benign, can alter the developing velocity field characteristics in laminar and turbulent impinging flows and thereby affect heat transfer distributions along the impinging surface. We consider steady laminar axisymmetric impinging jet flow of an inelastic Carreau fluid and analyze the effects of rheology on Nusselt number profiles at the impinging surface. Numerical calculations are presented for nozzle Reynolds numbers up to 750 and geometrical aspect ratios from $\frac{1}{4}$ to 4, while the chosen rheological parameter range allows for local viscosity variation of three orders of magnitude within high strain rate domains in the wall jet region of the flow. Our results demonstrate that substantial enhancement in heat transfer rates occur both in the impinging zone and in the wall jet. The calculations show that the local Nusselt number can be up to an order of magnitude larger, and the enhancement may persist for several nozzle diameters downstream of the stagnation point. The inclusion of even moderate temperature dependent viscosity behavior serves to augment the enhancement. We use the numerical results to identify those features of the developing velocity field that are primarily responsible for the observed heat transfer enhancement, and conclude that different underlying mechanisms are relevant in the impinging zone and in the wall jet region of the flow. Consideration also is given to the effects of viscous heating, which are shown to depend significantly on the geometrical aspect ratio. Suitably defined average Nusselt numbers are computed to quantify the extent of heat transfer enhancement as a function of the governing parameters describing fluid rheology and the developing flow field, and specific results are presented for impinging jet heat transfer with a 0.125% polyacrylamide solution.

1. Introduction

Steady impinging flows find wide applicability in situations requiring high heat and mass transport rates across the impinging surface. The archetypal axisymmetric impinging jet geometry is depicted in Fig. 1. For Newtonian fluids this is a classical problem with an extensive set of results and review articles [1–3] for both the axisymmetric geometry and its two dimensional counterpart, and recent papers have examined various additional complexities including transport in microscale jets [4–6], heat transfer from a spinning disk to an impinging jet [7], as well as anomalous features in the heat flux from a laminar premixed flame impinging on a flat surface [8]. Some previous publications have considered non-Newtonian impinging flow and heat transfer, particularly unconfined free surface jet flows, employing boundary layer approximations and seeking similarity solutions [9,10], while others have analyzed radial channel flow in the creeping flow limit to assess the effect of elasticity [11]. More recently, Zhao and Khayat [12] carefully considered the development and growth of the viscous boundary layer leading upto the hydraulic jump. Nevertheless, an exhaustive set of results and analyses comparable to the Newtonian case is not available for laminar impinging jet heat transfer with non-Newtonian fluids.

It is well-known that the details of the developing laminar velocity field in this geometry have a very significant influence on heat transfer along the impinging surface [1–3,13,14]. Even at moderate Reynolds numbers the dimensionless strain rate field in this flow varies over several orders, from a value of zero along the centerline far upstream of the nozzle exit to $O(10^3)$ along the boundary layer in the wall jet region. Thus heat transfer distributions for fluids with strain rate dependent viscosity are expected to display qualitative features not observed with Newtonian rheology, and a comparative assessment would be useful to delineate which features may be attributed solely to the rheology of the fluid. This prompted the earlier investigation [15] which employed the complete equations of motion and modeled non-Newtonian flow using the Carreau viscosity equation, but restricted the analysis to small departures from Newtonian rheology. These

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Nomenclature

Br	Brinkman number, $Br = \frac{(\hat{\mu}_0-\hat{\mu}_\infty)U^2}{k(T_1-T_0)}$ (–)
Cu	Carreau number, dimensionless inverse strain rate at which viscosity variation is important in the given flow, $Cu = \lambda\frac{U}{R}$ (–)
e	geometrical aspect ratio, $e = \frac{L}{R}$ (–)
h	heat transfer coefficient at the impinging surface $z = L$ ($\mathrm{W/m^2/K}$)
k	thermal conductivity of the fluid (W/m/K)
D	diameter of the nozzle (m)
L	distance from the nozzle exit to the impinging surface (m)
n	strength of viscosity dependence on strain rate (–)
Nu	Nusselt number, $Nu = \frac{hD}{k}$ (–)
Pr	Prandtl number of the fluid, based on $\hat{\mu}_0$ (–)
r	radial coordinate (m)
R	radius of the nozzle (m)
$Re, \hat{Re}$	Reynolds number of the flow, $Re = \frac{DU\rho}{\hat{\mu}_0}, \hat{Re} = \frac{DU\rho}{\hat{\mu}}$ (–)
T	temperature of the fluid (K)
U	average axial velocity of the fluid in the nozzle (m/s)
V_L	dimensionless length of a vortex, scaled with R (–)
z	axial coordinate (m)
$\vec{\vec{\Delta}}$	dimensionless rate of deformation tensor, $\Delta_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$, scaled with $\frac{U}{R}$ (–)
$\dot{\gamma}$	dimensionless scalar strain rate, $\dot{\gamma} = \sqrt{\frac{\vec{\vec{\Delta}}:\vec{\vec{\Delta}}}{2}}$, scaled with $\frac{U}{R}$ (–)
η	dimensionless axial coordinate, $\eta = \frac{z}{L}$ (–)
λ	inverse of strain rate threshold for viscosity variation (s)
$\hat{\mu}, \hat{\mu}_0$	fluid viscosity, viscosity at zero strain rate (kg/m/s)
θ	dimensionless temperature of the fluid, $\theta = \frac{T-T_0}{T_1-T_0}$ (–)
ρ	fluid density ($\mathrm{kg/m^3}$)
ξ	dimensionless radial coordinate, $\xi = \frac{r}{R}$ (–)

calculations showed that a distinct feature with shear thinning inelastic rheology is the occurrence of a maximum heat transfer rate within the impinging zone even at moderate aspect ratios, unlike the case with Newtonian fluids [14]. This non-Newtonian feature was noted even for very small departures from Newtonian rheology where the fully developed axial flow in the nozzle virtually was indistinguishable from the classical Newtonian Poiseuille parabolic profile. The maximum in heat transfer rate occurs due to a corresponding maximum in axial velocity as the flow approaches the impinging surface, this latter maximum arising as a result of the coupling between the developing radial strain rate profile and the relatively flat nozzle exit axial velocity profile for a shear thinning fluid. More recently, some publications have analyzed complex non-Newtonian flow field characteristics in impinging geometries from both experimental and computational viewpoints for laminar [16] and turbulent [17] regimes. Cavadas et al. [16] studied two dimensional laminar impinging flow of non-Newtonian fluids in a geometry confined by sloping inclined walls. In accordance with our previous calculations [15], their experiments indicated that the length of the separation vortex, which arises as the emerging flow from the nozzle turns past the corner and into the channel, is larger with non-Newtonian rheology.

In this work we again employ the Carreau rheological model since it has been used quite frequently to characterize inelastic non-Newtonian fluids [18,19]. The dimensionless Carreau constitutive equation is

$$\mu = \frac{\hat{\mu} - \hat{\mu}_\infty}{\hat{\mu}_0 - \hat{\mu}_\infty} = [1 + (Cu\dot{\gamma})^2]^{\frac{n-1}{2}} \tag{1}$$

where μ is the dimensionless viscosity, $\hat{\mu}_0$ the zero strain rate viscosity and $\hat{\mu}_\infty$ the viscosity as $\dot{\gamma} \to \infty$. Typically $\hat{\mu}_\infty \ll \hat{\mu}_0$ and we ignore it in our calculations except in Section 6 where we analyze a specific fluid and use its experimentally determined parameter values without further approximation. Cu is a dimensionless quantity that measures the inverse strain rate at which viscosity dependence is important in the given flow, and n is a measure of the strength of the viscosity dependence function. $\dot{\gamma}$ is the dimensionless scalar strain rate written in the usual way in terms of the rate of deformation tensor $\vec{\vec{\Delta}}$

$$\dot{\gamma} = \sqrt{\frac{\vec{\vec{\Delta}} : \vec{\vec{\Delta}}}{2}} \tag{2}$$

and the extra stress tensor is expressed as

$$\vec{\vec{\tau}} = -\mu\vec{\vec{\Delta}} \tag{3}$$

Our objective is to analyze the enhancement in heat transfer rate across the impinging surface $z = L$ for a laminar axisymmetric impinging jet with a Carreau fluid. We compute the Nusselt number distribution along the impinging surface and focus on a comparative analysis with the corresponding Newtonian result. These comparisons provide quantitative estimates of heat transfer enhancement, and an interpretation of these results in the context of the developing flow field reveals the specific local flow field features that are responsible for the enhancement. When the temperature range of interest is large enough the dependence of viscosity on temperature cannot be ignored, and we compute heat transfer results allowing for such dependence. In addition, we also analyze

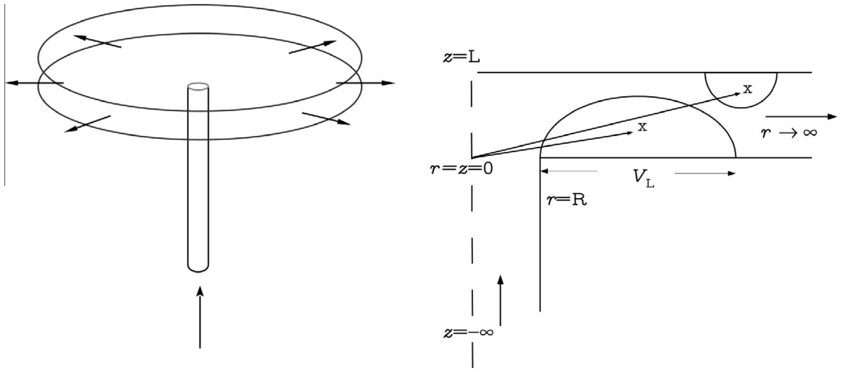

Fig. 1. The axisymmetric impinging flow geometry.