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Fatigue crack growth analysis in layered heterogeneous material systems using peridynamic approach

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ABSTRACT

In this study, the peridynamic fatigue model for a homogeneous material is extended to the layered heterogeneous material. Thermal residual stress and the corresponding stress intensity factor are calculated, within the framework of the peridynamic theory, by considering the cooling process using a pairwise force function caused by the thermal loading effect. To avoid overlapping of the cracked surfaces due to compressive thermal residual stress, the notion of short range force (Macek and Silling, 2007) is newly introduced. In addition, an auxiliary reference configuration is used to define the cyclic bond strain in the constricted material. The proposed approach is validated by performing an illustrative case study.

1. Introduction

The numerical prediction of the behaviors of a body containing discontinuities in the field of solid and fracture mechanics is a topic of continued research interest because of the difficulty in manipulating the spatial discontinuities that leads to the existence of spatial derivatives in the partial differential equation. Several methods based on the finite element method (FEM) have been proposed for dealing with this difficulty within the framework of the classical elasticity theory, such as the cohesive zone method [1–4] and the extended finite element method [5–8]. However, these methods have some inherent complexity in that the discontinuities cannot simultaneously be handled in the same partial differential equation without supplemental relations.

To overcome this issue, the peridynamic theory has recently been developed by reformulating the classical elasticity theory in order to treat the aforementioned discontinuities [9]. In the peridynamic theory, spatial derivatives are not required to analyze the behavior of the body containing the discontinuities such as cracks, because the partial differential equation resulting from the classical elasticity theory is changed into the associated integro-differential equation.

Although the peridynamic theory has been utilized during the past decade for addressing many scientific problems [10–16], problems concerning fatigue crack growth have rarely been addressed. Oterkus et al. [17] proposed a peridynamic fatigue model that can be applied to the bond-based peridynamic model. They assumed that the critical bond stretch, a peridynamic parameter related to the crack growth, is degraded exponentially with the cyclic load progress. Silling [18] proposed a more general peridynamic fatigue model that can be applied to any of the bond- and state-based peridynamic models [19] by introducing a notion of the remaining life of bonds. He showed that the fatigue crack growth model can be constructed based on Paris' law data obtained for a homogeneous material.

In the present paper, an efficient method for predicting the fatigue crack behavior of a heterogeneous material, such as a composite or a functionally graded material (FGM), subjected to a thermal loading condition is newly proposed by extending the peridynamic fatigue model developed by Silling [18]. The proposed method enables computation of the thermal residual stress (TRS) within the framework of the peridynamic theory. This study also proposes a way to avoid overlapping of the cracked surfaces under compressive TRS during the computation process. Furthermore, this study shows that the stress intensity factor (SIF) in the field of the TRS can be obtained by peridynamic simulations without the need for any other methods such as FEM. The present proposition is validated through performance evaluation and investigation of an illustrative case study.

2. Peridynamic theory for fatigue failure

2.1. Two-dimensional (2-D) bond-based peridynamic model

The general expression of the equation of motion in the bond-based peridynamic model, with the introduction of a Cartesian coordinate system, takes the following form [9]

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$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t)=\int_{H_{\mathbf{x}}}\mathbf{f}(\mathbf{u}(\mathbf{x}',t),\mathbf{u}(\mathbf{x},t),\mathbf{x}',\mathbf{x},t)\mathrm{d}V_{\mathbf{x}'}+\mathbf{b}(\mathbf{x},t), \tag{1}$$

where ρ is the density field, $\mathbf{x}$ is the position vector from the origin of the coordinate system to a material point, $\mathbf{x}'$ is a material point bonded to $\mathbf{x}$, t is time, $\mathbf{u}$ is the displacement, $\mathbf{f}$ is a pairwise force function defined as the force per unit volume squared exerted on $\mathbf{x}$ by the material point at $\mathbf{x}'$, and $\mathbf{b}$ is the body force density. The domain of the integral in Eq. (1) ($H_{\mathbf{x}}$) is the neighborhood of $\mathbf{x}$, which is a circular region centered at $\mathbf{x}$ and the radius of the neighborhood is denoted by the symbol δ called horizon.

The pairwise force function $\mathbf{f}$ can be assumed to be linearly dependent upon the bond stretch s as follows [16]:

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi})=\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{|\boldsymbol{\xi}+\boldsymbol{\eta}|}(c_1 s+c_2\theta),\quad |\boldsymbol{\xi}|<\delta, \tag{2}$$

where c_1 and c_2 are constants to be determined, s is the bond stretch, θ is the temperature difference, and $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are the relative position in the reference configuration and the relative displacement between $\mathbf{x}$ and $\mathbf{x}'$, respectively. Note that the direction of $\mathbf{f}$ is aligned with that of the relative position vector at the deformed state ($\boldsymbol{\xi}+\boldsymbol{\eta}$), and its magnitude is determined by the values of c_i ($i=1$ and 2), s, and θ.

The two constants c_1 and c_2 are determined by assuming the body under consideration to be isotropic, infinite, and homogeneous subjected to plane stress state [16].

$$c_1=\frac{6E}{\pi\delta^3(1-\nu)},\quad c_2=-\alpha c_1, \tag{3}$$

where E and ν are the elastic modulus and Poisson ratio, respectively, and α is the coefficient of thermal expansion (CTE).

The bond stretch s is defined by

$$s=\frac{|\boldsymbol{\xi}+\boldsymbol{\eta}|-|\boldsymbol{\xi}|}{|\boldsymbol{\xi}|}. \tag{4}$$

This bond stretch measures the expanded or contracted distance between two material points $\mathbf{x}$ and $\mathbf{x}'$ with respect to their distance at the reference configuration. Therefore, the sign of s determines whether the bond is in tension or compression. Note here that the Poisson ratio of the material used in the two-dimensional bond-based peridynamic model is always 1/3 because the bond stretch is independent of the direction of $\boldsymbol{\xi}$ owing to the material's isotropy.

2.2. Peridynamic fatigue model for a homogeneous material

In general, a peridynamic bond is broken when the bond is stretched to a degree that exceeds the critical stretch value predetermined from the fracture energy of the selected material [15]. On the other hand, the bond breakage in the peridynamic fatigue model is determined by calculating the remaining life of the bond whose direction is perpendicular to the axis of the mode-I fatigue crack [18].

The remaining life of the bond can be calculated by the following equations [18]:

$$\lambda(0)=1,\quad \frac{\mathrm{d}\lambda}{\mathrm{d}N}=-A_f\varepsilon^{m_f}, \tag{5}$$

where λ is the remaining life of the bond, N is the loading cycle, m_f is a positive parameter obtained directly from Paris' law data, A_f is a positive parameter requiring calibration using both Paris' law and peridynamic simulation data [18], and ε is the cyclic bond strain. This cyclic bond strain is defined by

$$\varepsilon=|s^+-s^-|, \tag{6}$$

where s^+ and s^- are the maximum and minimum bond stretches, respectively, calculated using Eq. (4). It should be noted that an auxiliary reference configuration ($|\boldsymbol{\xi}_{a,i}|$) at any material point i is used in calculating the cyclic bond strain in order to ensure that the signs of s^+ and s^- are positive. Here, the auxiliary reference configuration should not exceed the relative position of the material point at cooled state ($|\boldsymbol{\xi}_i+\boldsymbol{\eta}_i|$).

In the peridynamic fatigue model represented by Eq. (5), the bond breaks at the loading cycle when λ reaches 0.

2.3. Peridynamic fatigue model for a heterogeneous material

As mentioned in the Introduction, one of the novelties of the present study is the fact that the fatigue crack growth behavior of a heterogeneous material, such as a composite material or FGM, can be predicted by only using Paris' law data for the individual materials, without the need for additional experiments.

In the peridynamic theory, the heterogeneous material can be treated by setting the material properties of the bond between $\mathbf{x}$ and $\mathbf{x}'$ as their averaged values according to the following equations:

$$E(\mathbf{x},\mathbf{x}')=\frac{E_{\mathbf{x}}+E_{\mathbf{x}'}}{2},\quad \alpha(\mathbf{x},\mathbf{x}')=\frac{\alpha_{\mathbf{x}}+\alpha_{\mathbf{x}'}}{2},\quad A_f(\mathbf{x},\mathbf{x}')=\frac{A_{f_{\mathbf{x}}}+A_{f_{\mathbf{x}'}}}{2},\quad m_f(\mathbf{x},\mathbf{x}')=\frac{m_{f_{\mathbf{x}}}+m_{f_{\mathbf{x}'}}}{2}. \tag{7}$$

Furthermore, TRS caused by the difference in the CTE values between the layered heterogeneous materials should be considered when obtaining the effective SIF at the crack tip, which can be determined by the following weighted integral equation [20]

$$K_{Irs}(a)=\int_0^a h(x,a)\sigma_{rs}(x)\mathrm{d}x,\quad \text{when } a<\frac{3}{4}L, \tag{8}$$

where a is the crack length, h is the weight function universal for a given crack geometry [21], L is the total length of the body, and $\sigma_{rs}(x)$ is the TRS distribution prior to the crack growth perpendicular to the mode-I fatigue crack axis (x-axis). The peridynamic simulation results can be used to obtain the peridynamic stress tensor field [22], which is used to obtain the TRS distribution $\sigma_{rs}(x)$ required in Eq. (8).

3. Numerical simulation for an illustrative case study

To validate the proposed peridynamic fatigue model of heterogeneous materials, a series of numerical simulations are conducted on two types of layered (disc-shaped) compact single-core FGM specimens (S_1, S_2) [23]. The material denoted by 2124/SiC/10p refers to a composite fabricated by reinforcing Al-2124 with 10 vol% SiC particulate. Note that all of the simulations are conducted using the adaptive dynamic relaxation method [24].

The geometry, dimension, and material properties of these specimens are shown in Fig. 1, where the regions denoted by C and S are the core and surface layers, respectively. Following the experimental conditions [23], the specimens are assumed to be cooled down from 500 °C to room temperature (25 °C) in the present simulations.

Fig. 2(a) and (b) are the resulting TRS distributions of these specimens along the section A–A' right after the completion of the cooling process. These figures depict the hoop stress field that mostly affects the fatigue behavior of the specimens. To ensure the validity of these TRS distributions resulting from the peridynamic simulation, the TRS distributions obtained from a finite element method (FEM) [25] are also depicted in the same window. It should be noted that all the simulation results represent the first