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Predictive model for the spherical indentation of composite laminates with finite thickness

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ABSTRACT

The elastic response of composite laminates in contact with a stiff indenter is of immense importance in many applications. With the aim to further extend the knowledge on this subject, the contact problem of a stiff spherical indenter with a composite plate was simulated with a 2D axisymmetric model implemented on a commercial software. A parametric study was carried out numerically to study the effect of the different parameters on the force–indentation response. The results show that the indentation response of an orthotropic laminate is material independent and that it strongly depends on the laminate thickness. The lamina thickness has no significant effect on the force–indentation relation. Also, the effect of hole radius can be neglected for the hollow support experiment. Two simple equations were derived by normalizing and fitting FE results to predict the response of an orthotropic laminate of finite thickness in contact with stiff spherical indenter supported on two different boundary conditions (fully and hollow supported). The predictions of the derived equations were compared with experimental results and the available analytical models in the literature. The presented equations show an excellent correlation with the experimental investigations.

1. Introduction

Contact stresses are presented in many applications such as at the contact between a composite fan blade and its disk, or in rotating components such as ball bearings against a composite housing, or in applications in which a composite plate slides between a pair of rollers or in the impact of composite laminate with foreign body. Many such cases involve a composite material in contact with other surfaces, and they can be idealized as a contact problem between a stiff indenter and a composite plate of finite thickness supported on a fully or hollow rigid substrate.

In 1882, Hertz published for the first time a solution for the problem of frictionless contact between homogeneous elastic bodies under normal loading [1]. The elastic contact problem of a sphere and a homogeneous isotropic material is now well understood and has been summarized by Jonhson [2]. However, solutions for non-isotropic materials are much less available. Contact problems for transversely isotropic materials have been investigated by Green and Zerna [3], Leknitskii [4], Sveklo [5], Dahan and Zarka [6], Turner [7], among others.

One of the most widely used elastic contact laws for laminated plates was presented by Yang and Sun [8] based on Hertz theory. They assumed that the contact pressure and contact area could be obtained from the usual formulas for isotropic materials, but using the orthotropic modulus of the material in loading direction instead of the isotropic modulus of elasticity. However, this model solves the contact problem as a half space problem without taking into account the plate thickness effect. A general solution for contact loading of transversely isotropic materials was presented by Swanson [9] based on Turner theory [7]. The model showing that the contact parameters may be found from formulae similar to the classical Hertz law for isotropic materials, if the isotropic modulus is replaced by a combination of the transversely isotropic properties. Recently, Chen et al. [10] have proposed a modification on the model presented by Yang and Sun [8] to deal with the problem of contact between rigid indenter and laminated plate taking into account the plate thickness. Comparing the results of Yang and Sun model [8] and Chen et al. model [10] with the experimental results presented by Wu and Shyu [11], the predictions of Chen et al. model agree well with the experimental results for small indentation depth. However, the predictions of Yang and Sun model deviate from the experimental results at lower indentation depthes than the predictions of Chen et al. model.

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Efforts were done to correlate the contact force and the plate response by use of the functions and integral equations proposed by Green and Zerna [3]. A part from these efforts, Wu and Yen [12] and Chao and Tu [13] have used the solution proposed by Pagano [14] for approximating a point load and then numerically associated the resulting surface displacements to the indenter geometry. With such an approach they related the static indentation of a cross-ply laminate to the contact force exerted by a rigid sphere. Sankar [15] derived an approximate function for surface displacement in an orthotropic beam. This function was used to formulate the integral equation for the problem of smooth contact between a rigid cylinder and an orthotropic beam. Some researchers proposed a general method to approach the contact problem of anisotropic plates. This method is based on the combination of an elastic solution which describes the local contact phenomenon and the classical theory for the global response. For instance, Cairns and Lagace [16] have used the stress function proposed by Leknitskii [4] to study the thick composite laminates subjected to lateral loading. Their experimental results were found to be in good agreement with their predictions. As indicated by Swanson [17], using this method is difficult and it is difficult to assess the resulting accuracy.

On the other hand, some researchers use the Finite Element Method to predict the response of laminated plates subjected to indentation loads. Jung [18] introduced an axisymmetric FE model to study the response of laminated composite plates under static indentation load and a progressive damage failure between a circular plate and a rigid hemispherical tip indenter. Ye et al. [19] developed a 3D FE model to numerically predict the dent depth in composite laminates subjected to static indentation. Good agreement between the numerical results and the experimental results was found for larger indentation depths than those achieved in [18,20]. Gan et al. [21] introduced a 2D FE analysis to study the indentation response of composite laminate supported on rigid substrate and indented by cylindrical and spherical indenters. From this study, an empirical formula was introduced to predict the contact radius between the indenter and the composite laminate.

As a conclusion from this short review, the presented FE models in the literature have reported a good agreement with the experimental results but they were really time and money consuming techniques. However, the analytical models introduced in the literature save time and money although they are not accurate enough for all indentation cases.

Hence, the present paper introduces a parametric study on the indentation problem using a 2D axisymmetric FE model. This study aims to numerically derive a closed form equation for the elastic force–indentation depth relation, F–α, for composite laminates of finite thickness in contact with stiff spherical indenter of a given radius. Two different situations are considered: (i) the composite laminate is fully supported on a rigid substrate and (ii) the composite laminate is supported on a hollow substrate. Based on different analytical models presented in the literature without the thickness effect, two equations are numerically derived and validated with experimental results. The presented equations predict the response of composite laminates, F–α, under indentation load taking into account the effect of lamina and laminate thickness for both cases previously mentioned (full and hollow support).

2. Indentation response

2.1. Hertzian contact law for half space

As proposed by Swanson [9], the relation between the contact force (F) and indentation depth (α) for a half space laminated plate can be calculated with:

$$F = K\alpha^{1.5} \tag{1}$$

where K is the Hertzian contact stiffness for half space that can be approximated as $K = \frac{4}{3}\sqrt{r}E_T, r$ is the radius of the rigid spherical indenter and E_T an effective modulus for transversely isotropic materials.

Turner [7] introduced a method for calculating the effective modulus for transversally isotropic materials E_T as follows:

$$E_T = \frac{2}{\alpha_1\alpha_3} \tag{2}$$

The parameters α_1 and α_3 can be determined as:

$$\alpha_1 = \sqrt{\frac{\frac{E_{xx}}{E_{zz}} - \nu_{xz}^2}{1-\nu_{xy}^2}}$$

$$\alpha_2 = \frac{\frac{E_{xx}}{2G_{xz}} - \nu_{xz}(1+\nu_{xy})}{1-\nu_{xy}^2}$$

$$\alpha_3 = \frac{2(1-\nu_{xy}^2)}{E_{xx}}\sqrt{\frac{\alpha_1+\alpha_2}{2}}$$

where $E_{xx}, E_{zz}, G_{xz}, \nu_{xy}, \nu_{xz}$ are the three-dimensional effective elastic constants of transversely isotropic materials or composite laminate ($_z$ is the thickness direction and $_x$ and $_y$ are the in-plane directions).

As a result, the effective modulus, E_T, can be calculated if the three-dimensional effective elastic constants of the laminate are known. A number of methods has been developed to theoretically calculate the three dimensional properties of composite laminates based on the lamina properties [14,22–24]. However, as none of these methods is easy to be implemented, a simple model to compute the three dimensional properties of orthotropic laminates is proposed here. The equations of this model are summarized in Appendix A.

2.2. General indentation problem

The most important step in studying a problem is identifying the parameters that influence this problem. The parameters that influence a general indentation problem on a quasi-isotropic laminate can be summarized as follows:

$$F(\alpha, r, t, R, t_{ply}, \mathbf{E}_{ply}) \tag{3}$$

where F is the applied force (N), α is the indentation depth (mm), t is the laminate thickness (mm), R is the hole radius (mm) for the hollow supported case, t_{ply} is the lamina thickness (mm) and $\mathbf{E}_{ply}$ is the elastic properties of the ply.

If the ply thickness is small enough compared to other problem dimensions, the homogenized material properties can be used resulting in:

$$F(\alpha, r, t, R, \mathbf{E}_{Lam}) \tag{4}$$

where $\mathbf{E}_{Lam}$ includes the five independent material properties for transversely isotropic materials. In this case, the elastic problem is considered axisymmetric. Turner's [7] solution for half-space shows that this set of elastic properties can be reduced to the effective transverse modulus E_T defined in Eq. (2). Based on this, the response can be defined as:

$$F(\alpha, r, t, R, E_T) \tag{5}$$

According to Buckingham-π theorem [25], the latter equation can be normalized as follows: $\overline{F} = F/(E_T r^2), \bar{\alpha} = \alpha/r, \bar{t} = t/r$ and $\overline{R} = R/r$, resulting in:

$$\overline{F}(\bar{\alpha}, \bar{t}, \overline{R}) \tag{6}$$