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Instability of viscoelastic curved liquid jets with surfactants

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ABSTRACT

The prilling process is a common technique utilised in different applications in many industrial and engineering processes. Typically in such a process a liquid jet emerges from an orifice and thereafter breaks up into small spherical droplets of various sizes due to interfacial instabilities. As is common in many industrial applications the fluid used is often a mixture of various fluids and will typically contain polymers or other additives which will cause the fluid to behave like a non-Newtonian fluid. Furthermore, surfactants may be used in such processes to manipulate the size of the resulting droplets. In this paper, we model the fluid as a viscoelastic liquid and use the Oldroyd-B constitutive equation. We reduce the governing equations into a set of one-dimensional equations by using an asymptotic analysis and then we examine steady state solutions for viscoelastic rotating liquid jets with surfactants. We thereafter examine small perturbations to this steady state to investigate both linear and non-linear instability of the liquid jet.

1. Introduction

The fragmentation of viscoelastic liquid jets into droplets has many industrial applications such as sprays, fertilizers, ink jets and roll coating (see Eggers et al. [15], Middleman et al. [24], Basaran [5] and Mckinley [22] for reviews of the various applications). The widespread use of devices which utilise a liquid jet at ever decreasing length scales and with a need for greater accuracy motivates the need to understand the mechanisms of the capillary instability and break-up of non-Newtonian liquid jets. Rayleigh [29], who is credited with being the first to examine theoretically the instability of incompressible inviscid liquid jets, identified that liquid jets are unstable to waves which have a wavelength greater than their circumference. Moreover, it was shown that a most unstable wave exists, typically now referred to as the Rayleigh mode, which was responsible for breakup and droplet formation. The presence of viscosity within the fluid was considered by Weber [40], who found that the wavelength of the most unstable mode is increased by viscosity. Since these two pioneering works there has been a wealth of publications examining various features associated with instability of straight liquid jets including variations associated with jet structure, different liquids and linear and nonlinear growth (see Eggers [14]).

However, despite the abundance of literature on straight liquid jets relatively little has been developed in terms of liquid jets that are curved either by the action of some body force such a gravity or an electric field or by the application of a solid body rotation to the container from which the jet emerges. The latter case is particularly relevant to the industrial prilling process where curved jets are produced due to centrifugal instabilities. The work of Wallwork et al. [39] was the first to examine such a scenario and in that work the governing equations and linear stability of small disturbances along an inviscid curved liquid jet was considered. The authors also conducted some experiments for inviscid rotating liquid jets to complement their theoretical works and found agreement between the two. Wong et al. [42] conducted a series of experiments of viscous liquid jets to see the effect of different parameters on trajectory and droplet formation of the jet. Decent et al. [12] extended the analysis to include gravity in the examination of linear stability by Wallwork [38]. Moreover, the influence of viscosity on the trajectory and stability of the break-up of rotating liquid jets has been examined by Decent et al. [13]. Non-Newtonian fluids have been investigated by Uddin et al. [37] who used the power-law model to examine the linear instability of a rotating liquid. Uddin [36] examined non-linear temporal solutions of the governing equations for a rotating liquid jet by solving numerically, using a finite difference scheme based on the Lax Wendroff method, the governing equations based on the slender jet assumption.

Surfactants, which have a tendency to reduce the surface tension of a free surface, have been used in many free surface flows to alter the dynamics near the locality of breakup or rupture (see Xue et al. [43]). A numerical study has been used by Renardy

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[30] to find the break-up of Newtonian case and viscoelastic liquid jets for the Giesekus model and upper convected Maxwell model. The stability of viscoelastic jets has been discussed by Middleman [24]. Goldin et al. [17] compared the linear stability between inviscid, Newtonian and viscoelastic liquid jets. Mageda & Larson [21] used the Oldroyd-B model for ideal elastic liquids (called Boger fluids) for investigating the rheological behavior of polyisobutylene and polystyrene when the shear rates are low. Yildirim and Basaran [48] studied the threads of shear-thinning jets without surfactants and they found that shear-thinning plays an important role in determining the shape of the interface near breakup. They also investigated the break-up dynamics of liquid jets by using the Carrea model and compared the results with one-and two-dimensional models. The beads-on-string structure was studied by Clasen et al. [9] using the Oldroyd-B model. Ardekani et al. [4] investigated the dynamics of beads-on-string structure and filament thread for weakly viscoelastic jets by using the Giesekus constitutive equation and they compared the results to those of the Oldroyd-B model. Bhat et al. [6] examined formation of beads-on-a-string structures and found that there are sub-satellite beads in their experiments. Mckinley and Tripathi [23] and Anna and Mckinley [2] conducted experiments to observe various stages of the capillary break-up of viscoelastic liquid jets. An experimental study has been conducted by Zhang and Basaran [45] to investigate the effects of viscosity and surface tension. The majority of these studies of either Newtonian or non-Newtonian fluids has been examined without surfactants. However, many authors have studied the effects of adding small amounts of surfactants on straight liquid jets. For example, Whitaker [41] examined the instability of inviscid liquid jets with surfactants. The linear instability of viscous liquid jets and a surfactant has been carried out by Hansen et al. [18]. They have found that the growth rate decreases with including surfactants. Anshus [3] investigated theoretically the effect of surfactants on liquid jets in two cases which are compressible and incompressible. He found that the surfactants decrease the growth rate, especially in the case of incompressible liquid jets. Craster et al. [11] studied Newtonian liquid jets with surfactants by using a one-dimensional model. The case of weakly viscoelastic jets with surfactants has been examined by Zhang et al. [46] in a study in which they discussed the influence of the viscosity ratio, using the Oldroyd-B model. Timmermans & Lister [33] have used a nonlinear analysis to study a surfactant laden thread in inviscid liquid jets. They used a one-dimensional nonlinear model to examine the effect of the surfactant on the change of surface tension gradients. Ambravaneswaran & Basaran [1] also used a one-dimensional approximation model to investigate the nonlinear effects of insoluble surfactants on the break-up of stretching liquid bridges. Uddin [36] investigated the effects of surfactants on the instability of rotating liquid jets. He discovered that surfactants reduce the growth rate of liquid curved jets and hight rotation rates enhance the role of surfactants on break-up lengths. Stone & Leal [32] examined the break-up of liquid jets with surfactants by extending the work of Stone et al. [31] to include insoluble surfactants. Viscous liquid jets and soluble surfactants have been studied by Milliken et al. [25] and Milliken & Leal [26]. They observed that Marangoni stress decreased with increasing the viscosity and surfactant solubility.

In this paper, we will extend the work of Decent et al. [13] to investigate the break-up of viscoelastic liquid curved jets with surfactants by using the Oldroyd-B model. Furthermore, we reduce the governing equations into a set of non-dimensional equations to captures the dynamics of the break-up of low viscosity elastic solutions with insoluble surfactants. We also use an asymptotic approach to find steady state solutions and then examine a linear instability analysis on these solutions. In this study, we numerically solve these equations using a finite difference scheme based on the Lax Wendroff method to determine the break-up lengths and main and satellite droplet sizes.

2. Problem formulation

In this section we develop the governing equations which govern the dynamics of a liquid jet emerging from a rotating orifice with applications to the industrial prilling process. Since the derivation has been explained in length elsewhere (see Părău et al. [27,28]) we will not motivate the equations in great detail. In this regards, we assume that we have a large cylindrical container which has radius s_0 and rotates with angular velocity Ω. The liquid emerges from an orifice which is made in the side of this container. The radius of the orifice, a, is very small compared with the radius of the container. This problem is examined by choosing a coordinate system (X, Y, Z) rotating with the container, having an origin at the axis of the container. The position of the orifice is at $(s_0, 0, 0)$. Due to the rotation of the container, the liquid leaves the orifice following a curved trajectory. In this problem, which is the prilling process, we consider that the centripetal acceleration of the jet is very large compared with the force of gravity. Under this assumption one may assume the jet moves in the (X, Z) plane, so that the centerline can be described by coordinates $(X(s,t), 0, Z(s,t))$, where s is the arc-length along the middle of the jet which emerges from the orifice and t is the time (see Wallwork [38]). In any cross-section of the jet we also have plane polar coordinates (n, ϕ), which are the radial and azimuthal direction and have unit vectors which are $\mathbf{e_s}, \mathbf{e_n}, \mathbf{e}_\phi$ (see Decent et al. [12]). The velocity components for this problem are $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$, where $\boldsymbol{u}$ is the tangential velocity, $\boldsymbol{v}$ is the radial velocity and $\boldsymbol{w}$ is the azimuthal velocity.

Initially, we outline the continuity, momentum and constitutive equations of motion. Due to the surfactant concentration, we have a convection-diffusion equation along the liquid interface. We use the Oldroyd-B model for viscoelastic term. These equations therefore take the form

$$\begin{aligned}
&\nabla \cdot \boldsymbol{u} = 0,\\
&\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} - 2\boldsymbol{w}\times\boldsymbol{u} - \boldsymbol{w}\times(\boldsymbol{w}\times\boldsymbol{r}),\\
&\boldsymbol{\tau} = \mu_s(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T) + \boldsymbol{T},\\
&\frac{\partial \boldsymbol{T}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{T} - \boldsymbol{T}\cdot\nabla\boldsymbol{u}\cdot\boldsymbol{T} - (\nabla\boldsymbol{u})^T\cdot\boldsymbol{T} = \frac{1}{\lambda}\left(\mu_p\gamma - \boldsymbol{T}\right),
\end{aligned} \tag{1}$$

where u is the velocity in the form $\boldsymbol{u} = \boldsymbol{u}e_s + \boldsymbol{v}e_n + \boldsymbol{w}e_\phi$, ρ is the density of the fluid, p is the pressure, the angular velocity of the container is $\boldsymbol{w} = (0, w, 0), \mu_s$ is the viscosity of the solvent, $\boldsymbol{T}$ is the extra stress tensor that denotes to the term of viscoelastic, and μ_p is the viscosity of the polymer. The surfactant concentration along the jet is given by (see Stone & Leal [32] and Blyth & Pozrikidis [7])

$$\Gamma_t + \nabla_{\boldsymbol{s}}\cdot(\Gamma\boldsymbol{u_s}) + \Gamma(\nabla_{\boldsymbol{s}}\cdot\boldsymbol{n})(\boldsymbol{u}\cdot\boldsymbol{n}) = S(\Gamma, B_s) + \boldsymbol{D_s}\nabla_{\boldsymbol{s}}^2\Gamma, \tag{2}$$

where $\nabla_{\boldsymbol{s}} = (I - \mathbf{nn})\cdot\nabla$ is the surface gradient operator, $\boldsymbol{D_s}$ is the surface diffusivity of surfactant, $\boldsymbol{u_s} = (I - \mathbf{nn})\cdot\boldsymbol{u}$ is the surface or tangential velocity, and $\nabla_{\boldsymbol{s}}\cdot\mathbf{n} = 2\kappa$ where κ is the mean curvature of the free surface. The surfactant source term, S, takes absorption from the free surface into account, and acts as a function of surfactant concentration on the surface Γ and the bulk B_s. The third term on the left of (2) relates to the effect of normal forces on dilatation by expansion (see Blyth and Pozrikidis [7]). We consider in this study that the diffusivity of surfactant is small ($\boldsymbol{D_s} = 0$) and the surfactant is insoluble (*i.e.*, $S = 0$). For example, if surfactant with typical diffusivity $10^{-10} - 10^{-9}$ mm^2 s^{-1} (Tricot [34]) were added to the liquid-bridge experiments of Zhang, Padgett & Basaran [47], so that