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Partition of mixed-mode fractures in 2D elastic orthotropic laminated beams under general loading

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ABSTRACT

An analytical method for partitioning mixed-mode fractures on rigid interfaces in orthotropic laminated double cantilever beams (DCBs) under through-thickness shear forces, in addition to bending moments and axial forces, is developed by extending recent work by the authors (Harvey et al., 2014). First, two pure through-thickness-shear-force modes (one pure mode I and one pure mode II) are discovered by extending the authors' mixed-mode partition theory for Timoshenko beams. Partition of mixed-mode fractures under pure through-thickness shear forces is then achieved by using these two pure modes in conjunction with two thickness ratio-dependent correction factors: (1) a shear correction factor, and (2) a pure-mode-II energy release rate (ERR) correction factor. Both correction factors closely follow an elegant normal distribution around a symmetric DCB geometry. The principle of orthogonality between all pure mode I and all pure mode II fracture modes is then used to complete the mixed-mode fracture partition theory for a general loading condition, including bending moments, axial forces, and through-thickness shear forces. Excellent agreement is observed between the present analytical partition theory and numerical results from finite element method (FEM) simulations.

1. Introduction

An analytical method for partitioning mixed-mode fractures in orthotropic laminated double cantilever beams (DCBs) with rigid interfaces has been developed in the authors' recent work [1] by taking 2D elasticity into consideration in a novel way. In Ref. [1], the DCBs are under crack tip bending moments and axial forces. The present study extends Ref. [1] to include crack tip through-thickness shear forces which commonly occur in practice.

Some of the previous studies on the topic include Lu et al.'s work [2] where the finite element method (FEM) was used to investigate the effects of transverse shear on an orthotropic beam with a crack present. Orthotropic rescaling simplified the numerical analysis to three non-dimensional parameters. The total energy release rate (ERR) was obtained by using the J-integral and then the crack tip displacements allowed the total ERR to be partitioned into its individual mode I and II components. Wang and Qiao [3] extended the conventional 2D elasticity-based partition theory of Suo and Hutchinson [4] to take into account shear deformation by using first-order shear-deformable plate theory for a bimaterial interfacial crack. The total ERR was determined using the J-integral and related to the individual stress intensity factors by introducing an unknown parameter which, as the solution to an integral equation, must be determined from tabulated numerical data from a limited range of geometries and material configurations. Li et al. [5], also using Suo and Hutchinson's work [4], considered the effects of transverse shear loading on a crack between a layered, isotropic, linear elastic material. They asserted that it is not possible to use beam theories (even higher-order ones) to determine the shear component of the ERR because such an approach neglects the contribution of the deformation local to the crack-tip, which plays a crucial role. They concluded that a full elastic solution is required and used the FEM. They combined their results with Suo and Hutchinson's theory [4] for bending moments and axial forces to provide the ERR and mode partition for layered materials under general loading conditions. A disadvantage of this method, again, is its reliance on tabulated data. The effect of through-thickness shear forces on the fracture-mode partition of a DCB therefore still remains a crucial area of research.

The authors' partition theory based on classical beam theory [6–10] has given excellent predictions of mixed-mode fracture toughness for delamination in generally laminated composite beams [8–10] when compared against the experimental test data from some independent comprehensive testing [11–20]. In comparison, mixed-mode partition theories based on 2D elasticity

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Nomenclature

a	crack length in a DCB
b	width of a DCB
$c(\gamma)$	γ-dependent correction factor for ERR due to β_{P-2D} mode II, $G_{\beta_{P-2D}}$
E_L, E_T	in-plane Young's moduli in the longitudinal and transverse directions
G, G_I, G_{II}	total, mode I and II ERRs in 2D elasticity theory
G_T, G_{I-T}, G_{II-T}	total, mode I and II ERRs in Timoshenko beam theory
$G_{\theta_{P-T}}, G_{\beta_{P-T}}$	ERRs due to θ_{P-T} mode I and β_{P-T} mode II in Timoshenko beam theory
$G_{\theta_{P-2D}}, G_{\beta_{P-2D}}$	ERRs due to θ_{P-2D} mode I and β_{P-2D} mode II in 2D elasticity theory
$G_{\theta_{1-2D}}, G_{\beta_{1-2D}}$	ERRs due to θ_{1-2D} mode I and β_{1-2D} mode II in 2D elasticity theory
h_1, h_2, h	thicknesses of upper, lower and intact beams
k_s	interface spring stiffness
L	uncracked length of DCB
M_1, M_2	DCB tip bending moments on upper and lower beams
M_{1B}, M_{2B}	crack tip bending moments on upper and lower beams
N_1, N_2	DCB tip axial forces on upper and lower beams
N_{1B}, N_{2B}	crack tip axial forces on upper and lower beams
N_{1Be}	crack tip effective axial force on upper beam
P_1, P_2	DCB tip shear forces on upper and lower beams
P_{1B}, P_{2B}	crack tip shear forces on upper and lower beams
γ	thickness ratio, h_2/h_1
$\theta_{i-2D}, \beta_{i-2D}$	pure mode I and II (with $i = 1, 2, 3, 4$) in 2D elasticity theory
θ_{P-T}, β_{P-T}	shear force only pure mode I and II in Timoshenko beam theory
$\theta_{P-2D}, \beta_{P-2D}$	shear force only pure mode I and II in 2D elasticity theory
$\kappa(\gamma)$	γ-dependent through-thickness shear correction factor
μ_{LZ}	through-thickness shear modulus
ν_{LT}, ν_{TL}	in-plane Poisson's ratios
σ_n, τ_s	interface normal and shear stresses

Abbreviations

DCB	double cantilever beam
ERR	energy release rate
FEM	finite element method
VCCT	virtual crack closure technique

[1,4] have shown poor correlation with experimental results. It is, however, still an unanswered question as to which mixed-mode partition theory provides the most accurate results for brittle fracture subjected to other loading conditions such as fatigue or thermal loading. Therefore, it is still essential to develop mixed-mode partition theories based on 2D elasticity in order to provide a comprehensive set of tools for the understanding of interfacial fractures. This is the motivation of present work.

The format of this paper is as follows: Initially, in Section 2.1, the ERR partition based on Timoshenko beam theory [6–10] is extended to consider a 2D elastic orthotropic DCB with through-thickness shear forces alone at the crack tip. Then, in Section 2.2, a mixed-mode partition theory is established for the DCB under general loading conditions which include crack tip bending moments, axial forces and shear forces. Comparisons for the mixed-fracture mode partition theory are made against results from 2D FEM simulations for a combination of loading conditions in Section 3. Conclusions are drawn in Section 4.

2. Analytical development

2.1. Mixed-mode partitions with through-thickness shear forces alone at the crack tip

Fig. 1a shows a laminated DCB with its geometry and tip bending moments M_1 and M_2, axial forces N_1 and N_2, and through thickness shear forces P_1 and P_2. Fig. 1b shows the internal loads at the crack tip and the sign convention of the interface normal stress σ_n and shear stress τ_s. The total ERR and its partitions under crack tip through-thickness shear forces, P_{1B} and P_{2B}, are calculated from Timoshenko beam partition theory [6–10] as:

$$G_T = \frac{1}{2b^2 h_1 \kappa \mu_{LZ}} \left(P_{1B}^2 + \frac{P_{2B}^2}{\gamma} - \frac{(P_{1B}+P_{2B})^2}{1+\gamma} \right) = \{ P_{1B} \quad P_{2B} \} [C_T] \{ P_{1B} \quad P_{2B} \}^T \tag{1}$$

$$G_{I-T} = c_{I-T} \left(P_{1B} - \frac{P_{2B}}{\beta_{P-T}} \right)^2 \tag{2}$$

$$G_{II-T} = c_{II-T} \left(P_{1B} - \frac{P_{2B}}{\theta_{P-T}} \right)^2 \tag{3}$$

where

$$c_{I-T} = G_{\theta_{P-T}} \left(1 - \frac{\theta_{P-T}}{\beta_{P-T}} \right)^{-2}, \quad c_{II-T} = G_{\beta_{P-T}} \left(1 - \frac{\beta_{P-T}}{\theta_{P-T}} \right)^{-2} \tag{4}$$

$$G_{\theta_{P-T}} = \frac{1}{2b^2 h_1 \kappa \mu_{LZ}} \left(1 + \frac{\theta_{P-T}^2}{\gamma} - \frac{(1+\theta_{P-T})^2}{1+\gamma} \right) \tag{5}$$

$$G_{\beta_{P-T}} = \frac{1}{2b^2 h_1 \kappa \mu_{LZ}} \left(1 + \frac{\beta_{P-T}^2}{\gamma} - \frac{(1+\beta_{P-T})^2}{1+\gamma} \right) \tag{6}$$

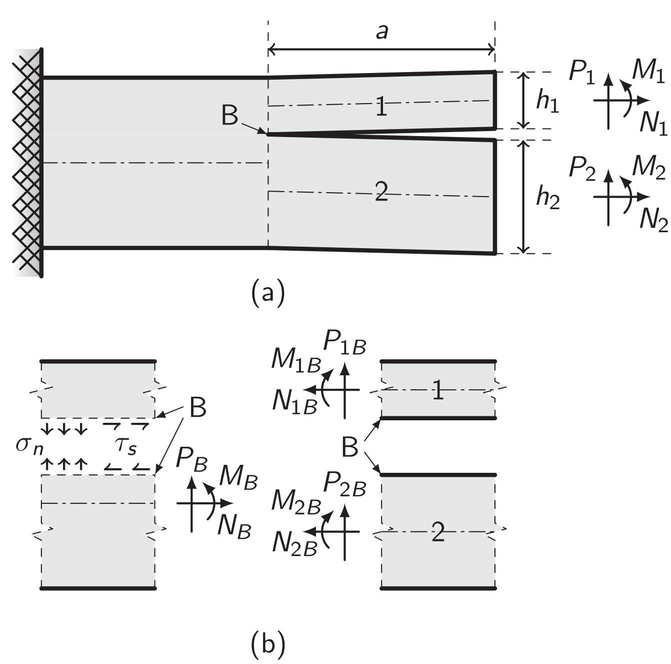

Fig. 1. A laminated DCB. (a) General description. (b) Details local to the crack tip.